Maximum Allowable Time Delay on Networked Control System Using Guaranteed Cost Method

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***Abstract*—In this paper, a robust filter is designed using guaranteed cost method to deal with the problem on networked control system (NCS). NCSs are modeled by neutral-type systems with differentiable time-delay uncertainties. The control and gain filter are derived from LMI feasible solution. A numerical example is provided for illustration of the proposed condition.**

***Keywords*—networked control system (NCS); guaranteed cost; LMI; neutral-type system**

I. INTRODUCTION

A classic feedback control system that is connected via a communication channel or a network, shared with other nodes outside the control system, is considered as a networked control system (NCS). NCSs gives benefits such as reducing wiring use, increasing mobility, giving easiness of monitoring systems and maintenance. However, due to the insertion of communication channels, control over network deals with some issues such as delay on network, packet drop, bandwidth allocation and scheduling, network security, fault tolerant control, and component integration [1], [2], [3].

Interesting techniques and results regarding guaranteed cost method have been studied in some varies field in NCSs such as asymptotic stabilization of uncertain nonlinear neutral time-delay system [4], uncertain NCS with time-varying delay using predictive control [3], T-S fuzzy controller for nonlinear neutral systems with time-delay [5]. In practical use, information data from sensors, actuators, and controllers are difficult to measure due to delay or intermittent loss in communication network, which lead to challenge on system analysis and design [6].

In this paper, we propose a filter design, a minimal order observer, to develop guaranteed cost control for uncertain NCSs time-delay. The uncertainties are norm-bounded with unknown initial state. The guaranteed cost control has an advantage in getting a lower bound of quadratic cost function on closed loop system. The stability on the LMIs using guaranteed cost method is found by deriving sufficient conditions for the existence of state feedback controller [7]. By solving the LMIs, we get the condition met, i.e. feedback control gain for the system be asymptotically stable and we also get the upper bound of maximum time delay as well.

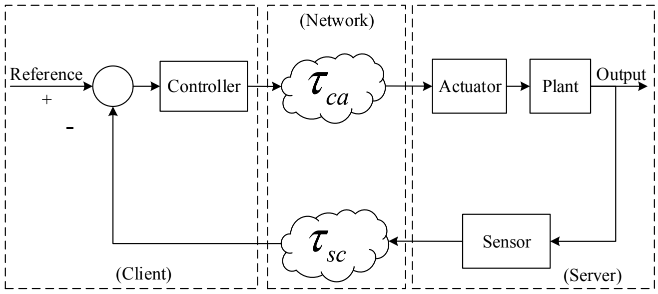

Fig. 1. NCS Block Diagram

II. SYSTEM DESCRIPTION

A typical NCS frame model with time-delay uncertainty is shown in Fig.1, where τ_{sc} is delay from sensor to controller (feedback time delay), and τ_{ca} is the delay from controller to actuator (command time delay). The controlled plant in Fig. 1 is a neutral-type plant, which can be modeled as follow:

Plant:

$$\begin{cases} \dot{x}(t) - G(t)\dot{x}(t-\tau(t)) = A_1 x(t) + A_2 x(t-d(t)) \\ \qquad + B_1 u(t) + B_2 w(t), \\ z(t) = C_1 x(\mathrm{t}) + Du(\mathrm{t}) + B_3 w(t), \\ y(t) = C_2 x(t), \\ x(\theta) = \psi(\theta), k \in \left[-\max\{d(t), \tau(t)\}, 0\right], \end{cases} \tag{1}$$

where $x \in \Re^n$, $y \in \Re^p$, $z \in \Re^d$, $u \in \Re^m$, $w \in \Re^l$ are the state, measured output, controlled output, control input, and disturbance input respectively.

Matrices G is assumed to have all of its eigenvalues inside the unit circle, $G(\lambda) < 1$. The time delay, $d(t)$ and neutral delay, $\tau(t)$ are assumed as unknown function of time and are differentiable continuously with rate:

$$0 \le d(t) \le d_m, 0 \le \tau(t) < \infty, \dot{d}(t) \le d_h < 1, \dot{\tau}(t) \le d_\tau < 1 \tag{2}$$

where d_m, d_h, and d_τ are positive constants and $\psi(.)$ is function of differentiable vector. We design filter that asymptotically stable on the system (1) as follow:

Filter system:

$$\begin{cases} \dot{\hat{x}}(t) - G(t)\dot{\hat{x}}(t-\tau(t)) = A_1(t)\hat{x}(t) + A_2(t)\hat{x}(t-d(t)) \\ \qquad + B_1(t)u(t) + L_i\left(y(t) - \hat{y}(\mathrm{t})\right), \\ \hat{y}(\mathrm{t}) = C_2(t)\hat{x}(t). \end{cases} \tag{3}$$

We propose output feedback control law as follow:

$$u(\mathrm{t}) = K_i\hat{x}(t), \tag{4}$$

where $\hat{x} \in \Re^n$, $\hat{y} \in \Re^n$ are filter state vector and output filter, also $K_i \in \Re^{m\times n}$ and $L_i \in \Re^{n\times m}$ are controller gain matrix and filter gain matrix. Let error state vector is modeled as follow:

$$e(t) = x(t) - \hat{x}(t). \tag{5}$$

Furthermore, we define an augmented state vector from system (1) and (5) as follow:

$$x_a(t) = \begin{bmatrix} x(t) \\ -e(t) \end{bmatrix}, \tag{6}$$

so that

Augmented system:

$$\begin{cases} \dot{x}_a(t) - \tilde{G}_a(t)\dot{x}_a(t-\tau(t)) = \tilde{A}_1(t)x_a(t) + \tilde{A}_2(t)x_a(t-d(t)) \\ \qquad + \tilde{B}_1(t)u(t) + \tilde{B}_2(t)w(t), \\ z(\mathrm{t}) = \tilde{C}_1 x_a(t) + D(t)u(t) + B_3 w(t) \end{cases} \tag{7}$$

where

$$u(t) = \tilde{K}(t)x_a(t). \tag{8}$$

We have corresponding augmented matrices as follow:

$$\tilde{A}_1(t) = \begin{pmatrix} A_1(t) & 0 \\ 0 & A_1(t) - L(t)C_2(t) \end{pmatrix},$$

$$\tilde{A}_2(t) = \begin{pmatrix} A_1(t) & 0 \\ 0 & A_1(t) \end{pmatrix}, \quad G(t) = \begin{pmatrix} G(t) & 0 \\ 0 & G(t) \end{pmatrix},$$

$$\tilde{B}_1(t) = \begin{pmatrix} B_1(t) \\ 0 \end{pmatrix}, \quad \tilde{B}_2(t) = \begin{pmatrix} B_2(t) \\ -B_2(t) \end{pmatrix},$$

$$\tilde{C}_1(t) = \begin{pmatrix} C_1(t) & 0 \end{pmatrix}, \quad \tilde{K}(t) = \begin{pmatrix} K(t) & K(t) \end{pmatrix}, \tag{9}$$

The following theorem will be used to obtain results in this paper using free-weighting matrix approach combined with an augmented Lyapunov-Krasovskii functional to investigate guaranteed cost control of NCSs.

Theorem 1. The neutral system in (1) and (2) (where $w(t) = 0$) is asymptotically stabilizable using feedback output control action (4), if there is constant matrices, namely P, Q, R, S and $\rho > 0$. Choose the Lyapunov-Krasovskii functional candidate to be:

$$V(t) = x^T(t)P_1x(t) + \int_{t-d(t)}^{t} x^T(s)Qx(s)ds + \int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds + \int_{-d_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S\dot{x}(s)dsd\theta, \tag{10}$$

where $P > 0$, $Q \geq 0$, $R \geq 0$, and $S > 0$, x_t denotes the translation operator acting on the trajectory, $x_t(\theta) = x(t+\theta)$ for (non-zero) interval $\theta \in [-d, 0]$. Lyapunov-Kravsovskii stability theorem implies:

$$\dot{V}(t,\phi) \leq -\rho\left(\|\phi\|^2\right). \tag{11}$$

Theorem 2. There exist constant $\gamma > 0$, so that the system (1)-(3) is asymptotically stabilizable using feedback output of control action (4), with H_∞ norm-bound γ if closed loop system (7) with $w(t) = 0$ is asymptotic stable and the subject to initial condition of state, the controlled output z should follow:

$$\|z\|_2 \leq \gamma\|w\|_2. \tag{12}$$

III. MAIN RESULTS

In order to find stabilization condition, first we construct closed-loop system from (7)-(8) for delay-dependent system as follow:

$$\begin{cases} \dot{x}_a(t) - \tilde{G}_a(t)\dot{x}_a(t-\tau(t)) = \tilde{A}_{1cl}(t)x_a(t) + \tilde{A}_2(t)x_a(t-d(t)) \\ \qquad + \tilde{B}_2(t)w(t), \\ z(\mathrm{t}) = \tilde{C}_{1cl}x_a(t) + B_3w(t) \end{cases} \tag{13}$$

where

$$\tilde{A}_{1cl} = \tilde{A}_1(t) + \tilde{B}_1(t)\tilde{K}(t),$$

$$\tilde{C}_{1cl} = \tilde{C}_1(t) + D_1(t)\tilde{K}(t). \tag{14}$$

From (10) and (13), define Lyapunov-Krasovskii functional candidate as follow:

$$V(t,x_a) = x_a^{\ T}(t)P_1x_a(t) + \int_{t-d(t)}^{t} x_a^{\ T}(s)Qx_a(s)ds + \int_{t-\tau(t)}^{t} \dot{x}_a^{\ T}(s)R\dot{x}_a(s)ds + \int_{-d_m}^{0}\int_{t+\theta}^{t} \dot{x}_a^{\ T}(s)S\dot{x}_a(s)dsd\theta, \tag{15}$$

for stability condition holds LMI equation as shown in (16), which is proven later.

Proof. To prove LMI (14), we need to determine $\dot{V}(t,x_a)$ along the trajectory (13).

$$\begin{aligned}\dot{V}(t,x_a) &= 2x_a^T(t)P_1\dot{x}_a(t) + x_a^T(t)Qx_a(t)\\ &-(1-\dot{d}(t))x_a^T(t-d(t))Qx_a(t-d(t))\\ &-(1-\dot{\tau}(t))\dot{x}_a^T(t-\tau(t))R\dot{x}_a(t-\tau(t))\\ &+\dot{x}_a^T(t)R\dot{x}_a(t) + d_m\dot{x}_a^T(t)S\dot{x}_a(t)\\ &+\int_{t-d_m}^{t} x_a^T(s)Sx_a(s)ds\end{aligned} \tag{17}$$

From (2), we also have:

$$0 \le \int_{t-d(t)}^{t} \dot{x}_a^T(s)S\dot{x}_a(s)ds \le \int_{t-d_m}^{t} \dot{x}_a^T(s)S\dot{x}_a(s)ds \tag{18}$$

to obtain (17), we can use Newton-Leibnitz formula for monovariant function f as follow:

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(s)ds = f(b(t))\dot{b}(t) - f(a(t))\dot{a}(t),$$

to provide

$$\int_{t-d(t)}^{t} \dot{x}_a(s)ds = x_a(t) - x_a(t-d(t)), \tag{19}$$

Let $P = \begin{bmatrix} P_1 & P_2 & P_3 \\ 0 & P_4 & P_5 \\ 0 & 0 & P_6 \end{bmatrix}$, and we can construct the first term on (17) to become:

$$2x_a^T(t)P_1\dot{x}_a^T(t) = 2\tilde{x}_a^T(t)P\begin{pmatrix} \dot{x}_a^T(t) \\ 0 \\ 0 \end{pmatrix} \tag{20}$$

where $\tilde{x}_a(t) = \begin{pmatrix} x_a(t) \\ \dot{x}_a(t) \\ x_a(t-d(t)) \end{pmatrix}$.

Substitute (13) and (19) into (20), now we have (21). Next, we can use Jensen's inequality for convex function (22) on (18), as follow

$$\varphi\left(\int_a^b f(x)dx\right) \le \frac{1}{b-a}\int_a^b \varphi\big((b-a)f(\mathrm{x})\big)dx, \tag{22}$$

to construct the last term on (17) so that:

$$-\int_{t-d(t)}^{t} \dot{x}_a^T(s)S\dot{x}_a(s)ds \le -\frac{1}{d_m}\left(\int_{t-d(t)}^{t} \dot{x}_a^T(s)ds\right)S.\left(\int_{t-d(t)}^{t} \dot{x}_a(s)ds\right), \tag{23}$$

substitute (21) and (23) into (17), then we add the new states $\int_{t-d(t)}^{t} \dot{x}_a(s)ds$ and $\dot{x}_a(t-\tau(t)$ into the derivative of Lyapunov-Krasovskii function and the condition of aysmptotic stabillity for $w(t)=0$, i.e.:

$$\dot{V}(t,x_a) \le \bar{x}^T(t,s)\Omega\bar{x}(t,s) < 0 \tag{24}$$

where $\bar{x}(t,s) = \begin{pmatrix} x_a(t) \\ \dot{x}_a(t) \\ \dot{x}_a(t-d(t)) \\ \int_{t-d(t)}^{t} \dot{x}_a(s)ds \\ \dot{x}_a(t-\tau(t)) \end{pmatrix}$ and

$$2x_a^T(t)P_1\dot{x}_a(t) = 2\tilde{x}_a^T(t)P.\begin{pmatrix} \dot{x}_a^T(t) \\ \tilde{A}_{1cl}(t)x_a(t) + \tilde{A}_2(t)x_a(t-d(t)) + \tilde{B}_2(t)w(t) - \dot{x}_a(t) + \tilde{G}_a(t)\dot{x}_a(t-\tau(t)) \\ x_a(t) - x_a(t-d(t)) - \int_{t-d(t)}^{t} \dot{x}_a(s)ds \end{pmatrix} \tag{21}$$

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & -P_3^T & P_2^T\tilde{G} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & -P_5^T & P_4^T\tilde{G} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & -P_6^T & 0 \\ -P_3 & -P_5 & -P_6 & -\frac{1}{d_m}S & 0 \\ \tilde{G}^T P_2 & \tilde{G}^T P_4 & 0 & 0 & -(1-d_\tau)R \end{pmatrix}$$

where

$\Omega_{11} = P_2^T\tilde{A}_{1cl} + \tilde{A}_{1cl}^T P_2 + P_3 + P_3^T + Q$,

$\Omega_{12} = \Omega_{21} = P_1 + P_4^T\tilde{A}_{1cl} + P_5^T - P_2$,

$\Omega_{13} = \Omega_{31} = \tilde{A}_2^T P_2 - P_3 + P_6^T$,

$\Omega_{22} = d_m S + R - P_4 - P_4^T$,

$\Omega_{23} = \Omega_{32} = \tilde{A}_2^T P_4 - P_5$,

$\Omega_{33} = -P_6 - P_6^T - (1-d_h)Q$.

Based on theorem 1, the required condition (11) is Ω must be negative definite.

Now, we include theorem 2 by using upper bound of $\gamma\|w\|_2$ for $L_2[0,\infty]$-z norm, to minimize the cost function from disturbance to controled for zero initial condition:

$$J = \int_0^\infty \left(\|z\|_2 - \|\gamma w\|_2\right) dt = \int_0^\infty \left(z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right) dt \quad (25)$$

then, we construct (25) become:

$$J = \int_0^\infty \left(z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t,x_a)\right) dt - V(\infty) \quad (26)$$

where

$$V(\infty) = x^T(\infty)P_1 x(\infty) + \lim_{t\to\infty}\left\{ \int_{t-h(t)}^{t} x^T(s)Qx(s)ds + \int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds + \int_{-d_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S\dot{x}(s)dsd\theta \right\}. \quad (27)$$

This is condition for any non-zeros $w(t) \in L_2[0,\infty]$, hence we add the state $w(t)$ to into the derivative of Lyapunov-Krasovskii function. The condition for aysmptotic stabillity, based on theorem 2 from (12), we have

$$J \le \int_0^\infty \xi^T(t,s)\bar{\Omega}\xi(t,s)dt - V(\infty) < 0 \quad (28)$$

where

$$\xi(t,s) = \begin{pmatrix} x_a(t) \\ \dot{x}_a(t) \\ \dot{x}_a(t-d(t)) \\ \int_{t-h(t)}^{t} \dot{x}_a(s)ds \\ \dot{x}_a(t-\tau(t)) \\ w(t) \end{pmatrix}.$$

and we have $\bar{\Omega}$ in (28) will be following the result as seen in equation (29), where

$\bar{\Omega}_{11} = P_2^T\tilde{A}_{1cl} + \tilde{A}_{1cl}^T P_2 + P_3 + P_3^T + \tilde{C}_{1cl}^T C_{1cl}^T + Q$,

$\bar{\Omega}_{12} = \bar{\Omega}_{21} = P_1 + P_4^T\tilde{A}_{1cl} + P_5^T - P_2$,

$\Omega_{13} = \Omega_{31} = \tilde{A}_2^T P_2 - P_3 + P_6^T$,

$\Omega_{22} = d_m S + R - P_4 - P_4^T$,

$\Omega_{23} = \Omega_{32} = \tilde{A}_2^T P_4 - P_5$,

$\Omega_{33} = -P_6 - P_6^T - (1-d_h)Q$.

Substitute (14) to (29) and set $P_4 = \beta P_2$. Now to find LMI (16), we form Schur complement by multiplying (29) both sides, pre and post-, with Δ below and its transpose.

$$\bar{\Omega} = \begin{pmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & -P_3^T & P_2^T\tilde{G} & \tilde{P}_2^T\tilde{B}_2 + \tilde{C}_{1cl}^T B_3 \\ \bar{\Omega}_{21} & \bar{\Omega}_{22} & \bar{\Omega}_{23} & -P_5^T & P_4^T\tilde{G} & P_4^T\tilde{B}_2 \\ \bar{\Omega}_{31} & \bar{\Omega}_{32} & \bar{\Omega}_{33} & -P_6^T & 0 & 0 \\ -P_3 & -P_5 & -P_6 & -\frac{1}{d_m}S & 0 & 0 \\ \tilde{G}^T P_2 & \tilde{G}^T P_4 & 0 & 0 & -(1-d_\tau)R & 0 \\ \tilde{B}_2^T P_2 + B_3^T\tilde{C}_{1cl} & \tilde{B}_2^T P_4 & 0 & 0 & 0 & -\gamma^2 I + B_3^T B_3 \end{pmatrix} \quad (29)$$

$$\Delta = \text{diag}\left(P_2^{-1}, P_2^{-1}, P_2^{-1}, P_2^{-1}, P_2^{-1}, I\right)$$

We introduce some modified variable such that:

$$X_2 = P_2^{-1}, \quad X_1 = X_2^T P_1 X_2, \quad X_3 = X_2^T P_3 X_2,$$
$$X_4 = X_2^T P_5 X_2, \quad X_5 = X_2^T P_6 X_2, \quad \bar{Q} = X_2^T Q X_2,$$
$$\bar{S} = X_2^T S X_2, \quad \bar{R} = X_2^T R X_2, \quad \tilde{U} = \tilde{K} X_2.$$

For closed-loop system (7) to be asymptotically stable, we use output feedback controller gain $\tilde{K} = \tilde{U} X_2^{-1}$.

IV. Numerical Example

Given a system as follow:

$$A_1 = \begin{pmatrix} 2 & 0.5 & 1 \\ -0.5 & -2 & 1 \\ 0 & 0 & 0.1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 0.1 & 0.2 \\ 0 & -1 & -0.1 \\ 0 & -0.1 & -2 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} -0.1 & 0.1 & -0.2 \\ -0.1 & -0.2 & 0.1 \\ 0 & 0 & -0.1 \end{pmatrix},$$

$C_2 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$, $C_1 = D = B_2 = 0$, we get upper bound time delay $d_m = 0.614$, $\alpha = 1.1625$ and given $L = \begin{pmatrix} 5.3943 \\ 1.6841 \\ 2.17 \end{pmatrix}$ and initial state as follow: $x(0) = \begin{pmatrix} 10 \\ 15 \\ 5 \end{pmatrix}$.

The result, as shown in Fig.2, shows that the system is asymptotically stable.

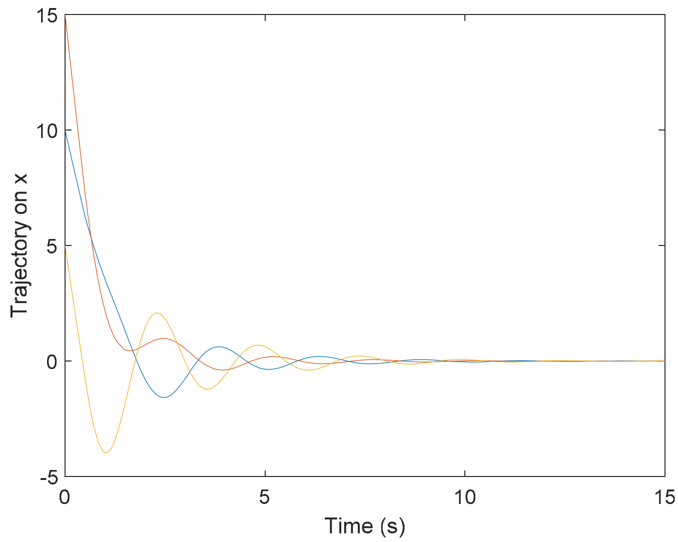

Fig. 2. Trajectory of state x, given initial condition, $x(0)$.

By solving the LMIs, we get the upper bound time delay on the system. The trajectory state of system leads to zero steady state, which is asymptotically stable as shown in the Fig. 2.

Discussion

For further research, we will implement guaranteed cost method on nanosatellite to control its attitude. We will make some adjustments on the system because nanosatellite consists of two or more reaction wheels as its driver, as well the controller.

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