Volume 9, Number 10, October 2015 ISSN 1881-803X

# ICIC Express Letters

*An International Journal of Research and Surveys*



Indexed by *Ei Compendex (Elsevier)*
*Scopus (Elsevier)*
*INSPEC (IET)*

Published by ICIC International
http://www.ijicic.org/icicel.htm

*ICIC Express Letters*
Volume **9**, Number **10**, October **2015**

ICIC International ©2015 ISSN 1881-803X
pp. 2717–2722

# A DC MOTOR-REACTION WHEEL CONTROL DESIGN VIA GUARANTEED COST OUTPUT FEEDBACK CONTROLLER OF UNCERTAIN NEUTRAL SYSTEMS

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Received February 2015; accepted May 2015

Abstract. *This paper presents the problem of guaranteed cost output feedback controller design for uncertain neutral systems. The output feedback stabilization usually leaves the mathematical algorithm which is not applicable, because of non-convexity and non-trivial computation task needs. To solve this kind of problems, we propose a sufficient condition for asymptotic stabilization by applying linear matrix inequalities (LMIs) feasible solutions. To show the usefulness of the proposed method, this method is applied to the DC motor of reaction wheel control design.*
**Keywords:** Guaranteed cost controller, Output feedback, Neutral systems, Linear matrix inequalities, DC motor-reaction wheel

## 1. Introduction.

Instability and bad performance can occur in a closed loop feedback control system with uncertainties. Therefore, considerable interests have been attracted to studies of robust controller design in recent decades. Moreover, it is desirable to design a controller which not only achieves the stability of the uncertain system but also guarantees an adequate level of performance. One of the approaches to solve is a guaranteed cost control method [1].

The stability and stabilization of neutral systems have been reported by many researches since the coverage of neutral system model using is wide, for example, modeling of chemical reactor and dynamic processes in steam and water pipes [2]. This paper presents the dc motor model of reaction wheels [3] in the form of an uncertain neutral system and implements the guaranteed cost output feedback control design to the model. The reaction wheels are used widely in satellite technologies for attitude because they have flexible torque capabilities, low power consumptions and good reliability [4]. Since the satellite is operated in atmospheric conditions, noises and attitude disturbances on its actuators very likely occur [5].

LMI approaches have been used to solve various stability and optimization problems. Also, many problems in system and control can be modeled or simplified by LMI methods [6]. Some reports on output feedback stabilization are [7,8] and references therein. However, these problems often leave the mathematical algorithm which is not applicable, because of non-convexity and non-trivial computation task needs [7]. In [7], this difficulty is solved by minimizing a positive real number iteratively to obtain a stabilizing static output feedback gain. In this work, we define an additional variable to reduce the complexity of inequalities and iteratively satisfy the inverse relations.

The contribution of this paper is the establishment of a design method of guaranteed cost output feedback controller for uncertain neutral systems. By minimizing the guaranteed cost function, an optimized upper bound is achieved. We adopt an iterative algorithm of observer based case [9] to achieve the stable closed loop condition. An application to the dc motor model of reaction wheel is shown to verify the effectiveness of the proposed method.

Outline of this paper is organized as follows. Section 2 describes the formulation of an uncertain neutral system. Section 3 provides the main results in LMIs, involving the output feedback control and robust output feedback control design. To verify the proposed method, Section 4 gives the simulation to DC motor-reaction wheel. Finally, Section 5 concludes this work.

**Notations**. Throughout the paper, the superscripts "$T$" and "$-1$" stand for matrix transpose and inverse, $\Re^n$ denotes the $n$-dimensional Euclidean space, $X > Y$ or $X \geq Y$ means that $X - Y$ is positive definite or semi-positive definite, $I$ is an identity matrix with appropriate dimensions, and * represents the symmetric elements in a symmetric matrix.

## 2. Problem Statement.

Consider a neutral system with uncertainties

$$\dot{x}(t) = A(t)x(t) + A_1(t)x(t-h(t)) + A_2(t)\dot{x}(t-\tau) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

$$x(t) = \psi(t) \tag{3}$$

where $A(t) = A + \Delta A(t)$, $A_1(t) = A_1 + \Delta A_1(t)$, $A_2(t) = A_2 + \Delta A_2(t)$, $B(t) = B + \Delta B(t)$, $h(t)$ is time varying delay, $\tau$ is neutral delay, $0 < h(t) < h$, $\dot{h}(t) \leq d$, $h$, $d$, and $\tau$ are known, $x \in \Re^n$ is the state vector, $y \in \Re^m$ is the measured output vector, $u \in \Re^m$ is control input, matrices $A$, $A_1$, $A_2$, $B$ are the real constants with appropriate dimension which represent the nominal of neutral time-delay system, $\psi$ is a given continuous vector-valued initial function, and $C$ is a known constant real-valued matrix with appropriate dimension. Matrices $\Delta A(t)$, $\Delta A_1(t)$, $\Delta A_2(t)$, $\Delta B(t)$ denote real norm-bounded matrix functions representing parameter uncertainties. It is assumed that

$$\Delta A(t) = D_A F_A(t) E_A,\ \Delta A_1(t) = D_{A_1} F_{A_1}(t) E_{A_1},\ \Delta A_2(t) = D_{A_2} F_{A_2}(t) E_{A_2},$$
$$\Delta B(t) = D_B F_B(t) E_B$$

which satisfy the inequalities

$$F_A^T(t)F_A(t) \leq I,\ F_{A_1}^T(t)F_{A_1}(t) \leq I,\ F_{A_2}^T(t)F_{A_2}(t) \leq I,\ F_B^T(t)F_B(t) \leq I,$$

where $D_A$, $E_A$, $D_{A_1}$, $E_{A_1}$, $D_{A_2}$, $E_{A_2}$, $D_B$, $E_B$ are constant real-valued known matrices with appropriate dimensions, and $F_A(t)$, $F_{A_1}(t)$, $F_{A_2}(t)$, $F_B(t)$ are real time-varying unknown continuous and deterministic matrices.

The problem considered here is to design an output feedback controller

$$u(t) = Ky(t) \tag{4}$$

to achieve an upper bound on the quadratic performance

$$J = \int_{-\infty}^{\infty} x^T(t)Qx(t) + u^T(t)Ru(t) \tag{5}$$

respect to systems (1)-(3), where $Q$ and $R$ are given symmetric positive definite matrices.

Before giving the main result, the following lemma is needed.

**Lemma 2.1.** *See [2]. Let $D$ and $E$ be matrices of appropriate dimensions, and $F$ be a matrix function satisfying $F^TF \leq I$, then for any positive scalar $\alpha$, the following inequality holds*

$$DFE + E^TF^TD^T \leq \alpha DD^T + \alpha^{-1}E^TE \tag{6}$$

**3. Main Results.** The main result of this study is given by Theorem 3.1.

**Theorem 3.1.** *If the following matrix inequalities optimization problem,* $\min\{\theta_1 + tr\theta_2 + tr\theta_3\}$ *subject to*

$$\begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 & \Pi_5 & \Pi_6 & \Pi_7 & \Pi_8 & \Pi_9 & 0 & \Pi_{10} & 0 & \Pi_{11} \\ * & -\beta^{-1}I & & & & & & & & & & & \\ * & & -\delta^{-1}I & & & & & & & & & & \\ * & & & -\varepsilon^{-1}I & & & & & & & & & \\ * & & & & -\rho^{-1}I & & & & & & & & \\ * & & & & & -\sigma^{-1}I & & & & & & & \\ * & & & & & & -R^{-1} & & & & & & \\ * & & & & & & & -I & & & & & \\ * & & & & & & & & -(1-d)P_2 & E_{A1}^T & & & A_1^T \\ 0 & & & & & & & & * & -(\varepsilon+\omega) & & & \\ * & & & & & & & & & & -P_3 & E_{A2}^T & A_2^T \\ 0 & & & & & & & & & & * & -(\rho+\nu) & \\ * & & & & & & & & * & & * & & -X_3 \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} -Y & P_1B \\ * & -I \end{bmatrix} < 0, \ \begin{bmatrix} -\theta_1 & x^T(0) \\ * & -P_1^{-1} \end{bmatrix} < 0, \ \begin{bmatrix} -\theta_2 & U^T \\ * & -P_2^{-1} \end{bmatrix} < 0, \ \begin{bmatrix} -\theta_3 & Z^T \\ * & -P_3^{-1} \end{bmatrix} < 0 \quad (8)$$

*where* $\Pi_1 = P_1A + A^TP_1 - Y + Q + P_2 + \beta^{-1}E_A^TE_A + (\delta^{-1} + \sigma^{-1})E_B^TE_B + \mu D_AD_A^T + \omega D_{A1}D_{A1}^T + \omega D_{A1}D_{A1}^T$, $\Pi_2 = P_1D_A$, $\Pi_3 = P_1D_B$, $\Pi_4 = P_1D_{A1}$, $\Pi_5 = P_1D_{A2}$, $\Pi_6 = (KC)^TE_B^T$, $\Pi_7 = (KC)^T$, $\Pi_8 = \left(B^TP_1 + KC\right)^T$, $\Pi_9 = P_1A_1$, $\Pi_{10} = P_1A_2$, $\Pi_{11} = (A + BKC)^T$ *has a set of solutions* $P_1 > 0$, $P_2 > 0$, $P_3 > 0$ *which satisfy the inverse relations* $X_1 = P_1^{-1}$, $X_2 = P_2^{-1}$, $X_3 = P_3^{-1}$, $\varepsilon_{inv} = \varepsilon^{-1}$, $\rho_{inv} = \rho^{-1}$, *then (4) is the guaranteed cost output feedback with an upper bound of the guaranteed cost*

$$V = x^T(0)P_1x(0) + \int_{-h(0)}^{0} x^T(s)P_2x(s)ds + \int_{-\tau}^{0} \dot{x}^T(s)P_3\dot{x}(s)ds \quad (9)$$

**Proof:** The closed loop system of (1) and (2) is

$$\dot{x}(t) - A_2(t)\dot{x}(t-\tau) = (A(t) + B(t)KC)x(t) + A_1(t)x(t-h(t)) \quad (10)$$

Consider the Lyapunov candidate

$$V = x^T(t)P_1x(t) + \int_{t-h(t)}^{t} x^T(s)P_2x(s)ds + \int_{t-\tau}^{t} \dot{x}^T(s)P_3\dot{x}(s)ds \quad (11)$$

Time derivative of (11) is

$$\begin{aligned} \dot{V} = {} & 2x^T(t)P_1\left((A(t) + B(t)KC)x(t) + A_1(t)x(t-h(t)) + A_2(t)\dot{x}(t-\tau)\right) + x^T(t)P_2x(t) \\ & - \left(1 - \dot{h}(t)\right)x^T(t-h(t))P_2x(t-h(t)) + \dot{x}^T(t)P_3\dot{x}(t) - \dot{x}^T(t-\tau)P_3\dot{x}(t-\tau) \end{aligned} \quad (12)$$

Introducing $\chi = [x(t) \ \ x(t-h(t)) \ \ \dot{x}(t-\tau)]^T$, we can rewrite (12) as

$$\dot{V} = \chi^T(t)\Psi\chi(t) - \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right) \quad (13)$$

where

$\Psi = [\Psi_1 \ \ \Psi_2 \ \ \Psi_3; * \ \ \Psi_4 \ \ \Psi_5; * \ \ * \ \ \Psi_6]$,

$\Psi_1 = P_1(A(t) + B(t)KC) + (A(t) + B(t)KC)^TP_1 + (KC)^TRKC + P_2 + Q + (A(t) + B(t)KC)^TP_3(A(T) + B(t)KC)$,

$\Psi_2 = P_1A_1(t) + (A(t) + B(t)KC)^TP_3A_1(t)$,

$\Psi_3 = P_1A_2(t) + (A(t) + B(t)KC)^TP_3A_2(t)$, $\Psi_4 = -\left(1 - \dot{h}(t)\right)P_2 + A_1^T(t)P_3A_1(t)$,

$\Psi_5 = A_1^T(t)P_3A_2(t)$, $\Psi_6 = -P_3 + A_2^T(t)P_3A_2(t)$.

Under condition $\Psi(t) < 0$, (13) leads to

$$\dot{V} < -\left(x^T(t)Qx(t) + u^T(t)Ru(t)\right) < 0 \tag{14}$$

for any $x(t) \neq 0$, and the closed loop system is asymptotically stable.

By using Lemma 2.1, for any scalars $\beta > 0$, $\delta > 0$, $\epsilon > 0$, $\rho > 0$, $\mu > 0$, $\sigma > 0$, $\omega > 0$, the following inequalities hold

$$2x^T(t)P_1\Delta Ax(t) \leq \beta x^T(t)P_1D_AD_A^TP_1x(t) + \beta^{-1}x^T(t)E_A^TE_Ax(t) \tag{15}$$

$$2x^T(t)P_1\Delta Bx(t) \leq \delta x^T(t)P_1D_BD_B^TP_1x(t) + \delta^{-1}x^T(t)E_B^TE_Bx(t) \tag{16}$$

$$\begin{aligned} 2x^T(t)P_1\Delta A_1x(t-h(t)) \leq\ & \epsilon x^T(t)P_1D_{A1}D_{A1}^TP_1x(t) \\ & + \epsilon^{-1}x^T(t-h(t))E_{A1}^TE_{A1}x(t-h(t)) \end{aligned} \tag{17}$$

$$2x^T(t)P_1\Delta A_2\dot{x}(t-\tau) \leq \rho x^T(t)P_1D_{A2}D_{A2}^TP_1x(t) + \rho^{-1}\dot{x}^T(t-\tau)E_{A2}^TE_{A2}\dot{x}(t-\tau) \tag{18}$$

$$2x^T(t)\Delta Ax(t) \leq \mu x^T(t)D_AD_A^Tx(t) + \mu^{-1}x^T(t)E_A^TE_Ax(t) \tag{19}$$

$$2x^T(t)(KC)^T\Delta Bx(t) \leq \sigma x^T(t)(KC)^TD_BD_B^TKCx(t) + \sigma^{-1}x^T(t)E_B^TE_Bx(t) \tag{20}$$

$$2x^T(t)\Delta A_1x(t-h(t)) \leq \omega x^T(t)D_{A1}D_{A1}^Tx(t) + \omega^{-1}x^T(t-h(t))E_{A1}^TE_{A1}x(t-h(t)) \tag{21}$$

$$2x^T(t)\Delta A_2\dot{x}(t-\tau) \leq vx^T(t)D_{A2}D_{A2}^Tx(t) + v^{-1}\dot{x}^T(t-\tau)E_{A2}^TE_{A2}\dot{x}(t-\tau) \tag{22}$$

Adopting [7], it is easy to find that

$$\begin{aligned} P_1(A+BKC) + (A+BKC)^TP_1 \leq\ & P_1(A+BKC) + (A+BKC)^TP_1 + (KC)^TKC \\ =\ & P_1A + A^TP_1 + \left(B^TP_1 + KC\right)^T\left(B^TP_1 + KC\right) \\ & - P_1BB^TP_1 \end{aligned} \tag{23}$$

To reduce the complexity, we define

$$P_1BB^TP_1 \leq Y \tag{24}$$

By using Schur complement [6], Equations (7) and (8) above are proved. □

Further, by integrating (14), we obtain the guaranteed cost

$$\begin{aligned} J &= \textstyle\int_0^\infty \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right)dt \\ &< x^T(0)P_1x(0) + \textstyle\int_{-h(0)}^0 x^T(s)P_2x(s)ds + \int_{-\tau}^0 \dot{x}^T(s)P_3\dot{x}(s)ds \\ &= J^* \end{aligned} \tag{25}$$

From (25), we derive the following relations

$$x^T(0)P_1x(0) < \theta_1, \tag{26}$$

$$\int_{-h(0)}^0 x^T(s)P_2x(s)ds = \int_{-h(0)}^0 tr\left(x^T(s)P_2x(s)\right)ds = tr\left(U^TP_2U\right) < tr(\theta_2), \tag{27}$$

$$\int_{-h(0)}^0 x^T(s)P_3x(s)ds = \int_{-h(0)}^0 tr\left(x^T(s)P_3x(s)\right)ds = tr\left(Z^TP_3Z\right) < tr(\theta_3) \tag{28}$$

Therefore, minimizing $\{\theta_1 + tr\theta_2 + tr\theta_3\}$ results in minimizing $J^*$.

4. **Simulation Result.** We implement the proposed method to the problem of reaction wheels for satellite attitude control [3]. The reaction wheel can be constructed based on DC motor shown in Figure 1.

We can derive the dynamic system equations both electrically and mechanically. The Kirchoff's voltage law of Figure 1 is written as

$$\Sigma V = 0,\ V - V_R - V_L - V_C = 0,\ V = iR_S + L\frac{di}{dt} + K_e\omega,\ \frac{di}{dt} = \frac{V}{L} - i\frac{R_S}{L} - \frac{K_e}{L}\omega \tag{29}$$

where $V$ is voltage, $i$ is current, $R_S$ is resistance, $L$ is inductance, $K_e$ is voltage constant, and $\omega$ is reaction wheel's angular velocity.

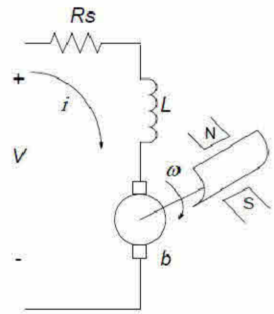


FIGURE 1. DC motor based reaction wheels

Next, sum of torque based on Newton's second law for rotational motion is equal to multiplication of inertia and angular acceleration $\dot{\omega}$.

$$\Sigma T = 0,\ J_m\dot{\omega} + b\omega + K_t i = 0,\ \frac{d\omega}{dt} = -\frac{b}{J_m}\omega - \frac{K_t}{J_m}i \tag{30}$$

Combination of (29) and (30) regarding to one of the axes $x$, $y$, $z$ yields the mathematical model of reaction wheels in the form of state space:

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_e}{L} \\ -\frac{K_t}{J_m} & -\frac{b}{J_m} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V,\ \omega = [0\ \ 1] \begin{bmatrix} i \\ \omega \end{bmatrix} \tag{31}$$

where $b$ is damping ratio, $J_m$ is inertia, and $K_e$ and $K_t$ are constants for electrical and mechanical torques. The parameter data are: $J_m = 9.6 \times 10^{-5}\text{kgm}^2$, $K_e = 0.0036\frac{\text{V}}{\text{rad}}/\text{s}$, $K_t = 3.4 \times 10^{-3}\frac{\text{Nm}}{\text{A}}$, $b = 2.4828 \times 10^{-8}\frac{\text{Nm}}{\text{rad}}/\text{s}$, $R_S = 22\text{ohm}$, $L = 590 \times 10^{-6}\text{H}$.

We can adjust (31) to (1) and (2) and because the delayed state and its derivative are not significant, i.e., motor motion is slow, we can assume that $A_1$ and $A_2$ are small and the uncertainties are in range 3%. Hence, nominal system data are

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{K_e}{L} \\ -\frac{K_t}{J_m} & -\frac{b}{J_m} \end{bmatrix},\ A_1 = A_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},\ B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix},\ C = [0\ \ 1],$$

and the other data are $h = 0.5$, $d = 0.4$, $\tau = 0.5$, $x(s) = \begin{bmatrix} 1-s \\ -1+s \end{bmatrix}$, $s \in [-1, 0]$, $U = \begin{bmatrix} \frac{19}{24} & -\frac{7}{24} \\ -\frac{7}{24} & \frac{7}{24} \end{bmatrix}$, $Z = \begin{bmatrix} \frac{1}{24} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{8} \end{bmatrix}$, $D_A = D_{A1} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$, $E_A = \begin{bmatrix} 0.3 & 0.3 \\ 0 & 0.3 \end{bmatrix}$, $E_{A1} = \begin{bmatrix} 0.3 & -0.3 \\ 0 & -0.3 \end{bmatrix}$, $D_{A2} = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $E_{A2} = \begin{bmatrix} 0.3 & -0.3 \\ 0 & 0.3 \end{bmatrix}$, $D_B = \begin{bmatrix} 0.3 & 0 \\ 0 & 0 \end{bmatrix}$, $E_B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R = 1$.

Then, the obtained results are

$P_1 = \begin{bmatrix} 0.0024 & 0.0000 \\ 0.0000 & 0.0269 \end{bmatrix}$, $X_1 = \begin{bmatrix} 417.0038 & -0.0072 \\ -0.0072 & 37.2394 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0.0563 & 0.0243 \\ 0.0243 & 0.0980 \end{bmatrix}$, $X_2 = \begin{bmatrix} 19.9079 & -4.9423 \\ -4.9423 & 11.4347 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0.0000 & -0.0001 \\ -0.0001 & 0.1095 \end{bmatrix}$, $X_3 = \begin{bmatrix} 22802 & 22 \\ 22 & 9 \end{bmatrix}$, $\varepsilon = 0.8811$, $\varepsilon_{inv} = 1.1350$, $\rho = 0.8833$, $\rho_{inv} = 1.1321$, $K = 2.9725$, $J^* = 0.1361$.

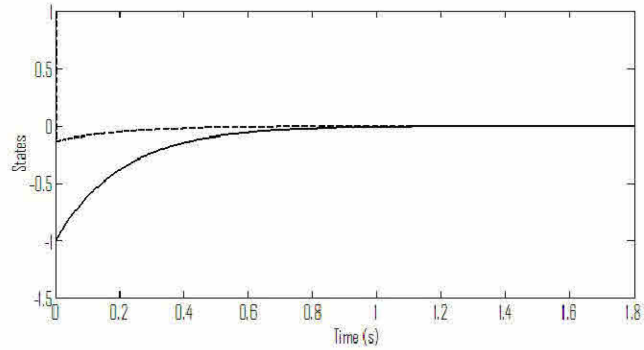


FIGURE 2. Trajectory of states $x_1$ (- -) and $x_2$ (—)

Trajectory of the states is figured in Figure 2. It is shown that the system converges to the stable condition. The results also satisfy and meet the inverse relations as follows

$$X_1 = P_1^{-1},\ X_2 = P_2^{-1},\ X_3 = P_3^{-1},\ \varepsilon_{inv} = \varepsilon^{-1},\ \rho_{inv} = \rho^{-1}$$

It guarantees the convergence of the optimization process such that the obtained output feedback gain can stabilize the closed loop system.

5. **Conclusions.** This paper discusses a guaranteed cost output feedback design for uncertain neutral systems. A sufficient condition for the guaranteed cost is derived on the basis of the LMI feasible solutions. The optimal cost is provided by minimizing the upper bound of the guaranteed cost. A simulation example of dc motor-reaction wheel control design is shown to illustrate the effectiveness of the proposed method. Future research will focus on the real system application, incorporated with Nano satellite research in Telkom University.

**Acknowledgment.** This research is funded by directorate general of higher education (DIKTI), Indonesia ministry of education and culture (Decision of the Director of Research and Community Service Number: 0263/E5/2014).

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