The 7th Electrical Power Electronics Controls Communications Informatics Seminar

EECCIS 2014

August 27-28, 2014. Universitas Brawijaya, Malang, Indonesia

2014 Electrical Power, Electronics, Controls, Communications, and Informatics Seminar (EECCIS)

IEEE INDONESIA SECTION

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2014 Electrical Power, Electronics, Controls, Communications, and Informatics Seminar (EECCIS)

IEEE Catalog Number : CFP1432Z-CDR
ISBN : 978-1-4799-6946-3

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An LMI Approach to Output Feedback Controller of Neutral Systems

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***Abstract*—This paper concerns on the problem of output feedback controller design for neutral systems. Generally speaking, not like a state feedback control problem, an output feedback control problem produces the non-convexity formulation in LMI so that it needs more complex computation. In this paper, we propose a sufficient condition for asymptotic stabilization by applying linear matrix inequalities (LMIs) feasible solutions. To solve the inverse relation, an iterative algorithm is applied. A numerical example is given to show the effectiveness of the proposed method.**

Keywords— output feedback, linear matrix inequalities, neutral system

I. INTRODUCTION

The problem of output feedback controller design has become the most area research in control theory and application for several decades [1].Since many problems in control engineering and application can be approached by output feedback synthesis and analysis then much attention has been devoted to this kind of interest.

Time-delay neutral system is the system with the dynamics depends on the delay of the state and its derivative. This system plays an important role in many engineering control applications. Those can be found easily in various fieldssuch as chemical reactor and dynamic process [2],[3]. Otherwise, system with dynamics which depends on only the delay of the state is the retarded-type system.

During last decades, LMIs approaches have been used to solve various stability and optimization problems. In [4], it is explained that many problems in system and control theory and application can be formulated and simplified by LMI methods. Some reports on output feedback control with LMI approach are [1], [5] and reference therein. However, problems of output feedback controller often leave the mathematical algorithm which not applicable, because of non-convexity and non-trivial which need computation tasks [1]. Therefore, this difficulty is solved by an iterative linear matrix inequalities scheme.

In this paper, design of output feedback controllers for neutral system is investigated.Non-convexity formulation on LMI is solved by adopting [1]. Because the inverse relations appear,an iterative algorithm is used [6].

The organization of this paper is as follows. Section II describes the problem statement. Main result and its proof are presented in Section III. The numerical example is given to show the effectiveness of the proposed method in Section IV, and as conclusion, Section V gives the summary of the paper.

Notations Throughout the paper, the superscripts ”T” and ” -1 ” stand for matrix transpose and inverse, $\Re^n$ denotesthe n-dimensional Euclidean space, $X > Y$ or $X \geqslant Y$ meansthat $X - Y$ is positive definite or semi-positive definite, I is anidentity matrix withappropriate dimensions, and * representsthe symmetric elements in a symmetric matrix.

II. PROBLEM STATEMENTS

Consider a neutral system

$$\dot{x} = A(t)x(t) + A_1(t)x\big(t - h(t)\big) + A_2(t)\dot{x}(t-\tau) + Bu(t) \quad (1)$$

$$y(t) = Cx(t), \quad (2)$$

$$x(t) = \psi(t) \quad (3)$$

where $h(t)$ is time varying delay, τ is neutral delay, $0 < \dot{h}(t) < h$, $\dot{h}(t) \leq d, h, d,$ and τ are known.

$x \in \mathbb{R}^n$is the state vector, $y \in \mathbb{R}^m$ is the measured output vector, matrices A, A_1, A_2, B are the real constants with appropriate dimensions which represent the nominal of neutral time-delay system, ψ is a given continuous vector-valued initial function, and$C$ is a known constant real-valued matrix with appropriate dimension.

The problem considered here is to design an output feedback controller

$$u(t) = Ky(t) \quad (4)$$

to achieve an upper bound on the quadratic performance

$$J = \int_{-\infty}^{\infty} x^T(t)Qx(t) + u^T(t)R\,u(t) \quad (5)$$

respect to system (1)-(3) where Q and R are given symmetric positive definite matrices.

III. MAIN RESULT

The main result of this study is given by Theorem1

Theorem 1. If the following matrix inequalities optimization problem; min $\{\theta_1 + tr\ \theta_2 + tr\ \theta_3\}$subject to

$$\begin{bmatrix} \Phi_1 & (KC)^T & \Phi_2 & P_1A_1 & P_1A_2 & \Phi_3 \\ * & -R^{-1} & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -(1-\dot{h}(t))P_2 & 0 & A_1^T \\ * & * & * & * & -P_3 & A_2^T \\ * & * & * & * & * & -X_3 \end{bmatrix} < 0 \quad (6)$$

$$\begin{bmatrix} -Y & P_1B \\ * & -I \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} -\theta_1 & x^T(0) \\ * & -P_1^{-1} \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} -\theta_2 & U^T \\ * & -P_2^{-1} \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} -\theta_3 & Z^T \\ * & -P_3^{-1} \end{bmatrix} < 0 \quad (10)$$

where

$$\boldsymbol{\Phi_1} = \boldsymbol{P_1A} + \boldsymbol{A^TP_1} - \boldsymbol{Y} + \boldsymbol{Q} + \boldsymbol{P_2},\ \boldsymbol{\Phi_2} = (\boldsymbol{B^TP_1} + \boldsymbol{KC})^{\boldsymbol{T}},\ \boldsymbol{\Phi_3} = (\boldsymbol{A} + \boldsymbol{BKC})^{\boldsymbol{T}}$$

has a set of solutions$\boldsymbol{P_1} > 0, \boldsymbol{P_2} > 0, \boldsymbol{P_3} > 0$ which satisfy the inverse relations [7],$\boldsymbol{X_1} = \boldsymbol{P_1^{-1}}$, $\boldsymbol{X_2} = \boldsymbol{P_2^{-1}}$, $\boldsymbol{X_3} = \boldsymbol{P_3^{-1}}$, then (4) is the optimal output feedback with cost function value

$$\boldsymbol{J} = \boldsymbol{x^T(0)P_1x(0)} + \int_{-\boldsymbol{h(0)}}^{\boldsymbol{0}} \boldsymbol{x^T(s)P_2x(s)ds} + \int_{-\boldsymbol{\tau}}^{\boldsymbol{0}} \boldsymbol{\dot{x}^T(s)P_3\dot{x}(s)ds} \quad \mathbf{(11)}$$

Proof of Theorem 1.

The closed loop system of (1)-(2)

$$\boldsymbol{\dot{x}(t)} - \boldsymbol{A_2\dot{x}(t-\tau)} = (\boldsymbol{A} + \boldsymbol{BKC})\boldsymbol{x(t)} + \boldsymbol{A_1x\big(t - h(t)\big)} \quad (12)$$

Consider the Lyapunov candidate

$$\boldsymbol{V(t)} = \boldsymbol{x^T(t)P_1x(t)} + \int_{\boldsymbol{t-h(t)}}^{\boldsymbol{t}} \boldsymbol{x^T(s)P_2x(s)ds} + \int_{\boldsymbol{t-\tau}}^{\boldsymbol{t}} \boldsymbol{\dot{x}^T(s)P_3\dot{x}(s)ds} \quad (13)$$

Time derivative of (14) is

$$\boldsymbol{V\dot{(}t)} = \boldsymbol{2x^T(t)P_1}\Big((\boldsymbol{A} + \boldsymbol{BKC})\boldsymbol{x(t)} + \boldsymbol{A_1x\big(t-h(t)\big)} + \boldsymbol{A_2\dot{x}(t-\tau)}\Big) + \boldsymbol{x^T(t)P_2x(t)} - \Big(\boldsymbol{1} - \boldsymbol{\dot{h}(t)}\Big)\boldsymbol{x^T\big(t-h(t)\big)P_2x\big(t-h(t)\big)} + \boldsymbol{\dot{x}^T(t)P_3\dot{x}(t)} - \boldsymbol{\dot{x}^T(t-\tau)P_3\dot{x}(t-\tau)} \quad (14)$$

Introducing

$$\boldsymbol{\chi(t)} = \begin{bmatrix} \boldsymbol{x(t)} \\ \boldsymbol{x(t-h(t))} \\ \boldsymbol{\dot{x}(t-\tau)} \end{bmatrix}$$

We can write (14) as

$$\boldsymbol{V\dot{(}t)} = \boldsymbol{\chi^T(t)\ \Psi\ \chi(t)} - (\boldsymbol{x^T(t)Qx(t)} + \boldsymbol{u^T(t)Ru(t)}) \quad (15)$$

where

$$\Psi = \begin{bmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ * & \Psi_4 & \Psi_5 \\ * & * & \Psi_6 \end{bmatrix}$$

$$\boldsymbol{\Psi_1} = \boldsymbol{P_1(A+BKC)} + \boldsymbol{(A+BKC)^TP_1} + \boldsymbol{P_2} + \boldsymbol{Q} + \boldsymbol{(KC)^TRKC} + \boldsymbol{(A+BKC)^TP_3(A+BKC)},$$

$$\boldsymbol{\Psi_2} = \boldsymbol{P_1A_1} + \boldsymbol{(A+BKC)^TP_3A_1},$$

$$\boldsymbol{\Psi_3} = \boldsymbol{P_1A_2} + \boldsymbol{(A+BKC)^TP_3A_2},$$

$$\boldsymbol{\Psi_4} = -\Big(\boldsymbol{1} - \boldsymbol{\dot{h}(t)}\Big)\boldsymbol{P_2} + \boldsymbol{A_1^TP_3A_1},$$

$$\boldsymbol{\Psi_5} = \boldsymbol{A_1^TP_3A_2}\ ,\ \boldsymbol{\Psi_6} = -\boldsymbol{P_3} + \boldsymbol{A_2^TP_3A_2},$$

Under condition $\boldsymbol{\Psi(t)} < 0$, (15) leads to

$$\boldsymbol{V\dot{(}t)} < -(\boldsymbol{x^T(t)Qx(t)} + \boldsymbol{u^T(t)Ru(t)}) < 0 \quad (16)$$

for any$\boldsymbol{x(t)} \neq \boldsymbol{0}$, and the closed loop system is asymptotically stable.

Note that $\boldsymbol{\Psi_1}$ is not an LMI form, but bilinear matrix inequality. Hence we adopt [1] to find

$$P_1(A+BKC) + (A+BKC)^TP_1 \le P_1(A+BKC) + (A+BKC)^TP_1 + (KC)^TKC = P_1A + A^TP_1 + (B^TP_1 + KCTBTP1+KC-P1BBTP1 \quad (17)$$

Now define

$$P_1BB^TP_1 \le Y \quad (18)$$

By using Schur complement [4], equations (6)-(10) above are proved.

Further, by integrating (16), we obtain

$$J=\int_0^\infty (x^T(t)Qx(t)+u^T(t)Ru(t))dt < x^T(0)P_1x(0) + \int_{-h(0)}^0 x^T(s)P_2x(s)ds + \int_{-\tau}^0 \dot{x}^T(s)P_3\dot{x}(s)ds = J^* \quad (19)$$

From (19), we define the following relations

$$x^T(0)P_1x(0) < \theta_1$$
$$\int_{-h(0)}^0 x^T(s)P_2x(s)ds = \int_{-h(0)}^0 tr(x^T(s)P_2x(s))ds = tr(U^TP_2U) < tr(\theta_2)$$
$$\int_{-h(0)}^0 x^T(s)P_3x(s)ds = \int_{-h(0)}^0 tr(x^T(s)P_3x(s))ds = tr(Z^TP_3Z) < tr(\theta_3) \quad (20)$$

Therefore, minimizing $\theta_1 + tr\,\theta_2 + tr\,\theta_3$ results in minimizing J^*

IV. Numerical Example

Consider a neutral system with following parameters

$$A=\begin{bmatrix}-3 & 0\\ -2 & 1\end{bmatrix}, A_1=\begin{bmatrix}0 & 2\\ 0 & 1\end{bmatrix}, A_2=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix},$$
$$B=\begin{bmatrix}-1\\ 2\end{bmatrix}, C=[0 \quad 1], h=0.5, d=0.4, \tau=0.5,$$
$$x(s)=\begin{bmatrix}1-s\\ -1+s\end{bmatrix}, s\in[-1,0], U=\begin{bmatrix}\frac{19}{24} & -\frac{7}{24}\\ -\frac{7}{24} & \frac{7}{24}\end{bmatrix},$$
$$Z=\begin{bmatrix}\frac{1}{24} & \frac{1}{16}\\ \frac{1}{16} & \frac{1}{8}\end{bmatrix}, Q=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}, R=2,$$

The obtained results are

$$K=-0.0619, J^*=1.1902,\ P_1=\begin{bmatrix}0.0959 & 0.0003\\ 0.0003 & 0.0958\end{bmatrix},$$
$$X_1=\begin{bmatrix}10.4310 & -0.0310\\ -0.0310 & 10.4375\end{bmatrix}, P_2=\begin{bmatrix}0.9968 & 0.0018\\ 0.0018 & 1.0559\end{bmatrix}$$
$$X_2=\begin{bmatrix}1.0032 & -0.0017\\ -0.0017 & 0.9471\end{bmatrix},$$
$$P_3=\begin{bmatrix}0.4345 & -0.1176\\ -0.1176 & 0.5502\end{bmatrix}, X_3=\begin{bmatrix}2.4427 & 0.5221\\ 0.5221 & 1.9290\end{bmatrix},$$

The results above satisfy the inverse relations

$$X_1=P_1^{-1},\ X_2=P_2^{-1},\ X_3=P_3^{-1}$$

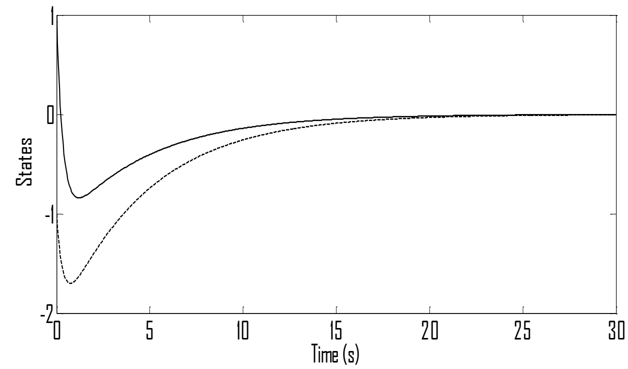

Fig. 1.Trajectory of states x_1(—) and x_2(- -).

Figure 1 shows the trajectory of states and figure 2 shows the output response. It is seen that thesystem is converged to the stable state

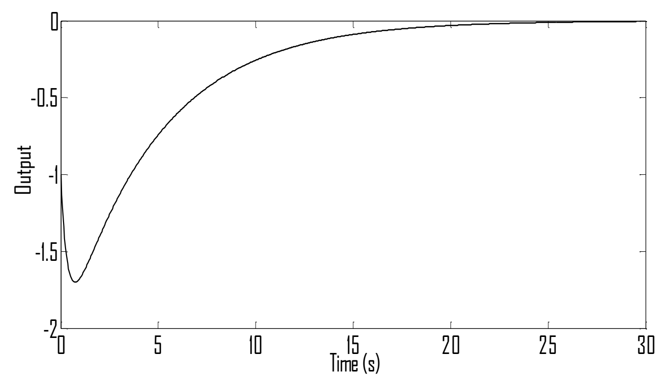

Fig. 2.Trajectory of output

The trajectory of states and output shows that the system responses converge to the stable conditions. The obtained variables satisfy the inverse relations show that the iterative algorithm can be implemented well.

V. Conclusion

This paper discusses a design of output feedback controller for neutral systems. A sufficient condition for the optimal cost function is derived on the basisof the LMI feasible solutions. The optimal cost is provided by minimizing the upper bound of the cost value. A numerical example is shown to illustrate the proposed method.

Acknowledgment

This research is funded by general directorate of higher education (DIKTI), Indonesia ministry of education and culture (Decision of the Director of Research and Community Service Number: 0263/E5/2014).

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