An LMI Approach to Guaranteed Cost Filter for Uncertain Neutral Systems with Time-varying Delay

Erwin Susanto[1] and Mitsuaki Ishitobi[2]
[1]*Engineering School, Telkom University, Bandung 40257, Indonesia*
[2]*Department of Mechanical Systems Engineering, Kumamoto University, Kumamoto 860855, Japan*
Email:[1]ews@ittelkom.ac.id, [2]mishi@kumamoto-u.ac.jp

Abstract— **This paper deals with the problem of robust guaranteed cost filter design for uncertain neutral systems with time-varying delay. The uncertainties are assumed to be norm bounded. We propose a sufficient condition for robust stability analysis and robust stabilization on the filter design, and the methods are derived in linier matrix inequalities (LMIs). The obtained result covers slow and fast time-varying delay cases by using Leibniz-Newton formula and some free-weighting matrices. A numerical example is given to illustrate the advantage of the proposed method.**

I. Introduction

DUE to the fact that the uncertainties and time-delay phenomena are frequently causing complex behaviors such as bad performance and instability in control systems and signal processing, the robust stability analysis and robust stabilization have drawn considerable attention in recent years. In the filter design, dynamics of the system states are estimated from actual output and the most faced-problems in this filtering scheme is that the output signals are corrupted by noise, disturbance, time-delay and parameter uncertainties [1]. Hence, it is desirable to design a controller or a filter which not only achieves the stability of the uncertain system but also guarantees an adequate level of performance. One of the possible approaches to solve this kind of problem is the guaranteed cost control or filter method [2], [3].

In the past decades, LMI approaches based on convex optimization solution have been used to analyze the stability of various dynamic systems. Many complex problems in system and control theory can be simplified by LMIs form. By using this approach, the feasible solutions can be found effectively [4].

The system whose dynamics depend on the delay of the state is the retarded-type system. Otherwise, the neutral time-delay system is the system with the dynamics depends on the delays of the state and its derivative. The type of this system can be found in many fields of engineering and technology such as chemical process modeling, networked control systems, robotic implementation in random environment, etc. Some reported results concerning on robust stability analysis and robust stabilization of the uncertain neutral time-delay systems are [5]-[7] and references therein.

Research on filter design considering stability analysis has started. In [3], robust guaranteed cost filtering problems for uncertain systems with time-varying delay was presented in LMI terms. The derivative of the time-varying delay was restricted to be less than one such that it leads to the conservative constrain. In [1], robust filtering problem for uncertain nonlinear systems was presented by using a polynomial Lyapunov function and a relaxation technique. In [8], H∞ filter design for systems with a time-varying delay was derived and extended to the systems with polytopic-type uncertainties. Robust H∞ filter design for uncertain systems with time-varying delays was proposed in [9]. In [10], a delay-dependent H∞ filtering design for linear neutral system was presented. However, the guaranteed cost filter design for uncertain neutral systems with time-varying delays has never been presented.

In this paper, we propose a design method of robust guaranteed cost filter for uncertain neutral systems with time-varying delay. The stabilization criteria are determined by a Lyapunov function and an LMI approach. Based on Leibniz-Newton formula and some free-weighting matrices, the derivative of time-varying delay is relaxed to any value. It enables this filter design appropriate for systems with fast and slow time varying delays. The upper bound of guaranteed cost of the considered systems is obtained. An illustrative example is given to demonstrate the merits of our proposed method.

Notations Throughout the paper, the superscripts ”T” and ” -1 ” stand for matrix transpose and inverse, $\Re^n$ denotes the n-dimensional Euclidean space, $X > Y$ or $X \geq Y$ means that $X-Y$ is positive definite or semi-positive definite, I is an identity matrix with appropriate dimensions, and * represents the symmetric elements in a symmetric matrix.

II. PROBLEM STATEMENT

Consider a continuous-time uncertain neutral system with time-varying delay

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - h(t)) + (A_n + \Delta A_n(t))\dot{x}(t - \tau) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

$$x(t) = \psi(t), t \in [-\max\{h, \tau\}, 0] \quad (3)$$

where $h(t)$ is a time-varying delay, τ is a neutral delay, $0 \le h(t) \le h$, $\dot{h}(t) \le d$, h and d are known values, $x(t) \in \Re^n$ is the state vector, $x(t) \in \Re^n$, $y(t) \in \Re^n$ is the measured output vector, $A, A_d, A_n \in \Re^n$ are the real constant matrices with appropriate dimensions which represent the nominal of neutral time-delay system, $x(t) = \psi(t)$ is a given continuous vector-valued initial function, C is a known constant real-valued matrix with appropriate dimension. Matrices $\Delta A(t), \Delta A_d(t), \Delta A_n(t)$ denote real norm-bounded matrix functions representing parameter uncertainties. It is assumed that

$$\Delta A(t) = D_A F_A(t) E_A, \Delta A_d(t) = D_{Ad} F_{Ad}(t) E_{Ad},$$
$$\Delta A_n(t) = D_{An} F_{An}(t) E_{An} \quad (4)$$

with satisfying the inequalities

$$F_A^T(t) F_A(t) \le I, F_{Ad}^T(t) F_{Ad}(t) \le I,$$
$$F_{An}^T(t) F_{An}(t) \le I$$

where $D_A, D_{Ad}, D_{An}, E_A, E_{Ad}, E_{An}$ are constant real-valued known matrices with appropriate dimensions, and $F_A(t), F_{Ad}(t), F_{An}(t)$ are real time-varying unknown continuous and deterministic matrices.

In this paper, we consider an asymptotically stable filter for system (1)-(3) above in the following forms

$$\dot{\hat{x}} = G\hat{x}(t) + A_d \hat{x}(t - h(t)) + A\dot{\hat{x}}(t - \tau) + Ky(t) \quad (5)$$

where G and K are filter variables.

Suppose error state vector and its derivative as follows

$$e(t) = x(t) - \hat{x}(t) \quad (6)$$

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) \quad (7)$$

The signal to be estimated and the upper bound of guaranteed cost to be minimized are given by

$$z(t) = Le(t) \quad (8)$$

$$J = \int_0^\infty z^T(t) z(t)\, dt \quad (9)$$

where $z(t)$ is the error state output and L is a given matrix.

In the sequel, some lemmas which will be useful for the proof are needed.

Lemma 2.1., (see [5]). Let D and E be matrices of appropriate dimensions, and F be a matrix function satisfying $F^T F \le I$

Then for any positive scalar α, the following inequality holds

$$DFE + E^T F^T D^T \le \alpha D D^T + \alpha^{-1} E^T E \quad (10)$$

Lemma 2.2., (see [11]). Related to the Leibniz-Newton formula, the following relation holds

$$2\left[w^T(t) N_1 + w^T(t - h(t)) N_2\right] \times \left[w^T(t) - w^T(t - h(t)) - \int_{t-h(t)}^{t} \dot{w}(s)\, ds\right] \quad (11)$$

where N_1, N_2 are free-weighting appropriate dimensioned matrices.

III. MAIN RESULTS

In this section, a sufficient condition is established for the existence of guaranteed cost filter and its upper bound of the guaranteed cost. The main result of this study is given by Theorem 3.1.

Theorem 3.1.

If the following matrix inequalities optimization problem;

$$\min\{\gamma_1 + \gamma_2 + tr(M_1 + M_2 + M_3 + M_4 + M_5)\}$$

subject to

$$\begin{bmatrix} \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_4 & \Lambda_5 & 0 & \Lambda_{14} & \Lambda_{15} & \Lambda_{16} & \Lambda_{17} & N_1 & 0 & \nabla \\ * & \Lambda_6 & 0 & \Lambda_7 & \Lambda_8 & 0 & 0 & 0 & \Lambda_{18} & \Lambda_{19} & N_2 & 0 & 0 \\ * & * & \Lambda_9 & 0 & 0 & 0 & 0 & 0 & \Lambda_{20} & \Lambda_{21} & 0 & 0 & 0 \\ * & * & * & \Lambda_{10} & \Lambda_{11} & \Lambda_{12} & X_3^T & \Lambda_{22} & 0 & 0 & 0 & N_1 & 0 \\ * & * & * & * & \Lambda_{13} & 0 & \Lambda_{23} & \Lambda_{24} & 0 & 0 & 0 & N_2 & 0 \\ * & * & * & * & * & -R_3 & \Lambda_{25} & \Lambda_{26} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Lambda_{27} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -P_3 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Lambda_{28} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Lambda_{29} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Lambda_{30} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -\Psi \end{bmatrix} < 0 \quad (12)$$

$$\begin{bmatrix} -\gamma_1 & x^T(0) \\ * & -P_{1inv} \end{bmatrix} < 0, \begin{bmatrix} -\gamma_2 & e^T(0) \\ * & -R_{1inv} \end{bmatrix} < 0, \begin{bmatrix} -M_1 & W_1^T \\ * & -P_{2inv} \end{bmatrix} < 0,$$
$$\begin{bmatrix} -M_2 & W_2^T \\ * & -R_{2inv} \end{bmatrix} < 0, \begin{bmatrix} -M_3 & W_3^T \\ * & -P_{3inv} \end{bmatrix} < 0, \begin{bmatrix} -M_4 & W_4^T \\ * & -R_{3inv} \end{bmatrix} < 0,$$
$$\begin{bmatrix} -M_5 & W_5^T \\ * & -P_{4inv} \end{bmatrix} < 0, \begin{bmatrix} -M_6 & W_6^T \\ * & -R_{4inv} \end{bmatrix} < 0, \quad (13)$$

where

$$\Phi = (A - G - KC),$$

$$\Lambda_1 = P_1 A + A^T P_1 + (\alpha_{1inv} + \alpha_{4inv} + \varepsilon_{1inv} + \varepsilon_{4inv}) E_A^T E_A + (\varepsilon_{7inv} + \varepsilon_{10inv}) E_A^T E_A + N_1 + N_1^T + P_2,$$

$$\Lambda_2 = P_1 A_d - N_1 + N_2^T, \Lambda_3 = P_1 A_n,$$

$$\Lambda_4 = \Phi^T R_1 + N_1 + N_1^T, \Lambda_5 = -N_1 + N_2^T,$$

$$\Lambda_6 = -(1-d) P_2 + (\alpha_{2inv} + \alpha_{5inv} + \varepsilon_{2inv}) E_{Ad}^T E_{Ad} + (\varepsilon_{5inv} + \varepsilon_{8inv} + \varepsilon_{11inv}) E_{Ad}^T E_{Ad} - N_2 - N_2^T,$$

$$\Lambda_7 = N_2, \Lambda_8 = N_2 + N_2^T,$$

$$\Lambda_9 = -P_3 + (\alpha_{3inv} + \alpha_{6inv} + \varepsilon_{3inv} + \varepsilon_{6inv}) E_{An}^T E_{An},$$

$$+(\varepsilon_{9inv}+\varepsilon_{12inv})E_{An}^T E_{An},$$
$$\Lambda_{10}=R_1G+G^TR_1+R_2+L^TL+N_1+N_1^T,$$
$$\Lambda_{11}=-N_1+N_2^T,\Lambda_{12}=R_1A_n+A_n^TR_1,$$
$$\Lambda_{13}=-(1-d)R_2-N_2+N_2^T,\Lambda_{14}=\Phi^TR_3,$$
$$\Lambda_{15}=\Phi^ThR_4,\Lambda_{16}=A^TP_3,\Lambda_{17}=hA^TP_4,\Lambda_{18}=A_d^TP_3,$$
$$\Lambda_{19}=hA_d^TP_4,\Lambda_{20}=A_n^TP_3,\Lambda_{21}=hA_n^TP_4,\Lambda_{22}=hX_4^T,$$
$$\Lambda_{23}=A_d^TR_3,\Lambda_{24}=hA_d^TR_4,\Lambda_{25}=A_n^TR_3,\Lambda_{26}=hA_n^TR_4,$$
$$\Lambda_{27}=-hR_4,\Lambda_{28}=-hP_4,\Lambda_{29}=-h^{-1}P_4,$$
$$\nabla_A=\begin{bmatrix}D_A & D_{Ad} & D_{An}\end{bmatrix},\nabla=\begin{bmatrix}P_1\nabla_A & P_3\nabla_A & hP_4\nabla_A\\ R_1\nabla_A & R_3\nabla_A & hR_4\nabla_A\end{bmatrix},$$
$$\Psi=diag\{\alpha_{1inv},\alpha_{2inv},\cdots,\alpha_{6inv},\varepsilon_{1inv},\varepsilon_{2inv},\cdots,\varepsilon_{12inv}\}$$

has a set of solution $P_1>0,R_1>0,P_2>0,R_2>0,P_3>0,$ $R_3>0,P_4>0,R_4>0,N_1>0,N_2>0,\gamma_1,\gamma_2,tr(M_1),tr(M_2),$ $tr(M_3),tr(M_4),tr(M_5),\alpha_{1inv},\alpha_{2inv},\alpha_{3inv},\alpha_{4inv},\alpha_{5inv},\alpha_{6inv},\varepsilon_{1inv},$ $\varepsilon_{2inv},\varepsilon_{3inv},\varepsilon_{4inv},\varepsilon_{5inv},\varepsilon_{6inv},\varepsilon_{7inv},\varepsilon_{8inv},\varepsilon_{9inv},\varepsilon_{10inv},\varepsilon_{11inv},\varepsilon_{12inv}$ which satisfy the relations $P_{1inv}=P_1^{-1},R_{1inv}=R_1^{-1},$ $P_{2inv}=P_2^{-1},R_{2inv}=R_2^{-1},P_{3inv}=P_3^{-1},R_{3inv}=R_3^{-1},P_{4inv}=P_4^{-1},R_{4inv}=R_4^{-1},$ and using variable changes $R_1G=X_1$, $R_3G=X_3$, $R_4G=X_4,R_1K=Y_1,R_3K=Y_3,R_4K=Y_4$, then the system (5) is a guaranteed cost filter with an upper bound of the guaranteed cost

$$J=x^T(0)P_1x(0)+e^T(0)R_1e(0)+\int_{-h}^{0}x^T(s)P_2x(s)\,ds$$
$$+\int_{-h}^{0}e^T(s)R_2e(s)\,ds+\int_{-\tau}^{0}\dot{x}^T(s)P_3\dot{x}(s)\,ds+\int_{-\tau}^{0}\dot{e}^T(s)R_3\dot{e}(s)\,ds$$
$$+\int_{-h}^{0}\int_{s}^{0}\dot{x}^T(\theta)P_4\dot{x}(\theta)\,d\theta\,ds+\int_{-h}^{0}\int_{s}^{0}\dot{e}^T(\theta)R_4\dot{e}(\theta)\,d\theta\,ds$$
$$=\gamma_1+\gamma_2+tr(M_1)+tr(M_2)+tr(M_3)+tr(M_4)+tr(M_5)$$
$$+tr(M_6) \qquad (14)$$

Remark 3.1: Since (12) and (13) have a constraint of the relationship of the inverse, an iterative algorithm via LMI approach, [12] is used based on a complementarity problem in [13].

Remark 3.2: The derivative of time-varying delay is relaxed by using Newton-Leibniz formula and some free-weighting semi-positive definite matrices N_1 and N_2. It is worthy for slow and fast time-varying delay cases [11].

Proof of Theorem 3.1.

Equations (1)-(3) and (5)-(7) yield the filtering error dynamics

$$\begin{bmatrix}\dot{x}(t)\\ \dot{e}(t)\end{bmatrix}=\begin{bmatrix}\Phi_1 & \Phi_2 & \Phi_3 & 0 & 0 & 0\\ \Phi_4 & \Delta A_d(t) & \Delta A_n(t) & G & A_d & A_n\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h(t))\\ x(t-\tau)\\ e(t)\\ e(t-h(t))\\ e(t-\tau)\end{bmatrix} \qquad (15)$$

where

$$\Phi_1=A+\Delta A(t),\Phi_2=A_d+\Delta A_d(t),\Phi_3=A_n+\Delta A_n(t),$$
$$\Phi_4=(A+\Delta A(t))-G-KC$$

Define a candidate of Lyapunov function

$$V(t)=w^T(t)\begin{bmatrix}P_1 & 0\\ 0 & R_1\end{bmatrix}w(t)+\int_{t-h(t)}^{t}w^T(t)\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t)\,dt$$
$$+\int_{t-\tau}^{t}\dot{w}^T(s)\begin{bmatrix}P_3 & 0\\ 0 & R_3\end{bmatrix}w(s)\,ds$$
$$+\int_{t-h(t)}^{t}\int_{s}^{t}\dot{w}^T(s)\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}w(s)\,d\theta\,ds \qquad (16)$$

where $w(\cdot)=\begin{bmatrix}x^T(\cdot) & e^T(\cdot)\end{bmatrix}^T$

The time derivative of (16) along to (15) is calculated as

$$\dot{V}(t)=2w^T(t)\begin{bmatrix}P_1 & 0\\ 0 & R_1\end{bmatrix}\dot{w}(t)+w^T(t)\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t)$$
$$-(1-\dot{h}(t))w^T(t-h(t))\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t-h(t))$$
$$+\dot{w}^T(t)\begin{bmatrix}P_3 & 0\\ 0 & R_3\end{bmatrix}\dot{w}(t)-\dot{w}^T(t-\tau)\begin{bmatrix}P_3 & 0\\ 0 & R_3\end{bmatrix}\dot{w}(t-\tau)$$
$$+h\dot{w}^T(t)\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}\dot{w}(t)-\int_{t-h(t)}^{t}\dot{w}^T(s)\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}\dot{w}(s)\,ds \qquad (17)$$

Since $\dot{h}(t)\le d$, this following inequality is bounded

$$w^T(t)\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t)-(1-\dot{h}(t))w^T(t-h(t))\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t-h(t))$$
$$\le w^T(t)\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t)-(1-d)w^T(t-h(t))\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t-h(t)) \qquad (18)$$

We need to relax $-(1-d)<0$ such that the LMIs have the less conservative results. It is done by using Leibniz-Newton formula and some free-weighting matrices as the following descriptions.

Consider Leibniz-Newton formula

$$w(t)=w(t-h(t))+\int_{t-h(t)}^{t}\dot{w}(s)\,ds \qquad (19)$$

To obtain a less conservative stabilization criterion, Lemma 2.2 is recalled.

Further, add Lemma 2.2 to (16)

$$\dot{V}(t)=2w^T(t)\begin{bmatrix}P_1 & 0\\ 0 & R_1\end{bmatrix}\dot{w}(t)+w^T(t)\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t)$$
$$-(1-\dot{h}(t))w^T(t-h(t))\begin{bmatrix}P_2 & 0\\ 0 & R_2\end{bmatrix}w(t-h(t))$$
$$+\dot{w}^T(t)\begin{bmatrix}P_3 & 0\\ 0 & R_3\end{bmatrix}\dot{w}(t)-\dot{w}^T(t-\tau)\begin{bmatrix}P_3 & 0\\ 0 & R_3\end{bmatrix}\dot{w}(t-\tau)$$
$$+h\dot{w}^T(t)\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}\dot{w}(t)-\int_{t-h(t)}^{t}\dot{w}^T(s)\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}\dot{w}(s)\,ds$$

$$+2\left[w^T(t)N_1+w^T(t-h(t))N_2\right]\times\left[w^T(t)-w^T(t-h(t))-\int_{t-h(t)}^{t}\dot{w}(s)\,ds\right] \tag{20}$$

From (20), it is notified that

$$-\int_{t-h(t)}^{t}\dot{w}^T(s)\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}\dot{w}(s)\,ds-2\left[w^T(t)N_1+w^T(t-h(t))N_2\right]$$

$$\int_{t-h(t)}^{t}\dot{w}(s)\,ds=-\int_{t-h(t)}^{t}\left\{w^T(s)N+\dot{w}^T(s)\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}\right\}$$

$$\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}^{-1}\left\{N^Tw(s)+\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}\dot{w}(s)\right\}ds$$

$$+\int_{t-h(t)}^{t}w^T(s)N\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}^{-1}N^Tw(s)\,ds$$

where $N=\left[N_1^T \quad N_2^T\right]^T$.

We can note that because $\begin{bmatrix}P_4 & 0\\ 0 & R_4\end{bmatrix}^{-1}$ is positive, then the first term on the right-hand must be negative.

Now, introduce $p(t)$ and $\Omega(t)$ to rewrite (20)

$$\dot{V}(t)=p^T(t)\Omega(t)p(t)-z^T(t)z(t) \tag{21}$$

where

$$p(t)=\begin{bmatrix}x(t)\\ x(t-h(t))\\ \dot{x}(t-\tau)\\ e(t)\\ e(t-h(t))\\ \dot{e}(t-\tau)\end{bmatrix},\ \Omega(t)=\begin{bmatrix}\Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 & \Omega_6\\ * & \Omega_7 & \Omega_8 & \Omega_9 & \Omega_{10} & \Omega_{11}\\ * & * & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15}\\ * & * & * & \Omega_{16} & \Omega_{17} & \Omega_{18}\\ * & * & * & * & \Omega_{19} & \Omega_{20}\\ * & * & * & * & * & \Omega_{21}\end{bmatrix},$$

$$\begin{aligned}
\Omega_1 &= P_1(A+\Delta A(t))+(A+\Delta A(t))^TP_1+P_2\\
&\quad+(A+\Delta A(t))^TP_3(A+\Delta A(t))+hN_1P_4^{-1}N_1^T\\
&\quad+(A+\Delta A(t)-G-KC)^TR_3(A+\Delta A(t)-G-KC)\\
&\quad+h(A+\Delta A(t))^TP_4(A+\Delta A(t))+N_1+N_1^T\\
&\quad+h(A+\Delta A(t)-G-KC)^TR_4(A+\Delta A(t)-G-KC),\\
\Omega_2 &= P_1(A_d+\Delta A_d(t))+(A+\Delta A(t))^TP_3(A_d+\Delta A_d(t))\\
&\quad+(A+\Delta A(t)-G-KC)^TR_3\Delta A_d(t)\\
&\quad+h(A+\Delta A(t))^TP_4(A_d+\Delta A_d(t))-N_1+N_2^T\\
&\quad+h(A+\Delta A(t)-G-KC)^TR_4\Delta A_d(t),\\
\Omega_3 &= P_1(A_n+\Delta A_n(t))+(A+\Delta A(t))^TP_3(A_n+\Delta A_n(t))\\
&\quad+(A+\Delta A(t)-G-KC)^TR_3\Delta A_n(t)\\
&\quad+h(A+\Delta A(t))^TP_4(A_n+\Delta A_n(t))\\
&\quad+h(A+\Delta A(t)-G-KC)^TR_4\Delta A_n(t),\\
\Omega_4 &= (A+\Delta A(t)-G-KC)^T(R_1+R_3G)\\
&\quad+h(A+\Delta A(t)-G-KC)^TR_4G+N_1+N_1^T,\\
\Omega_5 &= (A+\Delta A(t)-G-KC)^T(R_3+hR_4)A_d,\\
\Omega_6 &= (A+\Delta A(t)-G-KC)^T(R_3+hR_4)A_n,\\
\Omega_7 &= -(1-d)P_2+(A_d+\Delta A_d(t))^TP_3(A_d+\Delta A_d(t))\\
&\quad+\Delta A_d(t))^T(R_3+hR_4)\Delta A_d(t)+hN_2P_4^{-1}N_2^T\\
&\quad+h(A_d+\Delta A_d(t))^TP_4(A_d+\Delta A_d(t))-N_2-N_2^T,\\
\Omega_8 &= (A_d+\Delta A_d(t))^T(P_3+hP_4)(A_n+\Delta A_n(t))\\
&\quad+\Delta A_d(t)^T(R_3+hR_4)\Delta A_n(t),\\
\Omega_9 &= \Delta A_d(t)^T(R_1+R_3G+hR_4G)+N_2,\\
\Omega_{10} &= \Delta A_d(t)^T(R_3+hR_4)A_d+N_2+N_2^T,\ \Omega_{11}=\Delta A_d(t)^T(R_3+hR_4)A_n,\\
\Omega_{12} &= (A_n+\Delta A_n(t))^T(P_3+hP_4)(A_n+\Delta A_n(t))-P_3\\
&\quad+\Delta A_n(t)^T(R_3+hR_4)\Delta A_n(t),\\
\Omega_{13} &= \Delta A_n(t)^T(R_1+R_3G+hR_4G),\Omega_{14}=\Delta A_n(t)^T(R_3+hR_4)A_d,\\
\Omega_{15} &= \Delta A_n(t)^T(R_3+hR_4)A_n,\\
\Omega_{16} &= R_1G+G^TR_1+R_2+G^T(R_3+hR_4)G+N_1+N_1^T+hN_1R_4^{-1}N_1^T,\\
\Omega_{17} &= R_1\Delta A_d(t)+\Delta A_d^T(t)R_1+G^T(R_3+hR_4)A_d+L^TL-N_1+N_2^T,\\
\Omega_{18} &= R_1A_n+A_n^TR_1+G^T(R_3+hR_4)A_n,\\
\Omega_{19} &= -(1-d)R_2+A_d{}^T(R_3+hR_4)A_d-N_2-N_2^T+hN_2R_4^{-1}N_2^T,\\
\Omega_{20} &= A_d{}^T(R_3+hR_4)A_n,\Omega_{21}=A_n{}^T(R_3+hR_4)A_n-R_3
\end{aligned}$$

Under the condition

$$\Omega(t)<0 \tag{22}$$

equation (21) leads to

$$\dot{V}(t)<-z^T(t)z(t)<0 \tag{23}$$

for any $z(t)\neq 0$ and the filtering error dynamics is asymptotically stable.

Applying Lemma 2.1. to (21) for any positive scalars $\alpha_1,\alpha_2,\ldots,\alpha_6,\alpha_1^{-1},\alpha_2^{-1},\ldots,\alpha_6^{-1},\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_{12},\varepsilon_1^{-1},\varepsilon_2^{-1},\ldots,\varepsilon_{12}^{-1}$ and using Schur Complement of Boyd et al. [4], inequality (12) is obtained.

Further, integrating (23) from 0 to t and as t tends to the infinity

$$\dot{V}(t)<\int_0^{\infty}-z^T(t)z(t)\,dt<0 \tag{24}$$

implies

$$\begin{aligned}
J &= \int_0^{\infty}z^T(t)z(t)\,dt\\
&< x^T(0)P_1x(0)+e^T(0)R_1e(0)+\int_{-h}^{0}x^T(s)P_2x(s)ds\\
&\quad+\int_{-h}^{0}e^T(s)R_2e(s)ds+\int_{-\tau}^{0}\dot{x}^T(s)P_3\dot{x}(s)ds\\
&\quad+\int_{-\tau}^{0}\dot{e}^T(s)R_3\dot{e}(s)ds+\int_{-h}^{0}\int_{s}^{0}\dot{x}^T(\theta)P_4\dot{x}(\theta)\,d\theta\,ds\\
&\quad+\int_{-h}^{0}\int_{s}^{0}\dot{e}^T(\theta)R_4\dot{e}(\theta)\,d\theta\,ds=J^*
\end{aligned} \tag{25}$$

where J^* denotes the guaranteed cost and the following equations are defined,

$$\int_{-h}^{0}x(s)x^T(s)ds=W_1W_1^T,$$

$$\int_{-h}^{0}e(s)e^T(s)ds=W_2W_2^T,$$

$$\int_{-\tau}^{0}\dot{x}(s)\dot{x}^T(s)ds=W_3W_3^T,$$

$$\int_{-\tau}^{0}\dot{e}(s)\dot{e}^T(s)ds=W_4W_4^T,$$

$$\int_{-h}^{0}\int_{s}^{0}\dot{x}(\theta)\dot{x}^T(\theta)\,d\theta\,ds=W_5W_5^T,$$

$$\int_{-h}^{0}\int_{s}^{0}\dot{e}(\theta)\dot{e}^T(\theta)\,d\theta\,ds = W_6W_6^T \qquad (26)$$

It is obvious that (26) corresponds to (13).

IV. An Illustrative Example

Consider a system (1)-(3) with

$$A=\begin{bmatrix}-1 & 0.5\\ 0.5 & -2\end{bmatrix}, A_d=\begin{bmatrix}0.1 & -0.1\\ 0.3 & 0.2\end{bmatrix}, A_n=\begin{bmatrix}0.1 & -0.1\\ 0.1 & 0.1\end{bmatrix},$$

$$D_A=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{bmatrix}, E_A=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, D_{Ad}=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{bmatrix},$$

$$E_{Ad}=\begin{bmatrix}0 & 0.1\\ 0 & 0.1\end{bmatrix}, D_{An}=\begin{bmatrix}0 & 0.1\\ 0 & 0.1\end{bmatrix}, E_{An}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$

$$C=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.5\end{bmatrix}, L=\begin{bmatrix}1\\ 1\end{bmatrix}, x(s)=\begin{bmatrix}1-s\\ -1+s\end{bmatrix},$$

$$\hat{x}(s)=\begin{bmatrix}1-2s\\ -0.5+s\end{bmatrix}, s\in[-1,0]$$

A. A slow time-varying delay

In the case of a slow time-varying delay: $h = 0.5$, $d = 0.1$, $\tau = 0.5$, we have the following results.

From (26), it is easily obtained

$$W_1=\begin{bmatrix}0.8635 & -0.2146\\ -0.2146 & 0.4956\end{bmatrix}, W_2=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.5\end{bmatrix},$$

$$W_3=\begin{bmatrix}0.5000 & -0.5000\\ -0.5000 & 0.5000\end{bmatrix}, W_4=\begin{bmatrix}0.7071 & 0\\ 0 & 0\end{bmatrix},$$

$$W_5=\begin{bmatrix}0.5000 & 0.5000\\ 0.5000 & 0.5000\end{bmatrix}, W_6=\begin{bmatrix}0.7071 & 0\\ 0 & 0\end{bmatrix}$$

Applying Theorem 3.1, we have

$$P_1=\begin{bmatrix}7.5102 & -3.3755\\ -3.3755 & 7.1683\end{bmatrix}, P_2=\begin{bmatrix}3.5413 & -1.3047\\ -1.3047 & 1.3047\end{bmatrix},$$

$$P_3=\begin{bmatrix}3.2226 & -2.3732\\ -2.3732 & 2.9535\end{bmatrix}, P_4=\begin{bmatrix}2.9583 & 2.1014\\ 2.1014 & 2.7873\end{bmatrix},$$

$$R_1=\begin{bmatrix}1.0168 & 0.0197\\ 0.0197 & 2.3349\end{bmatrix}, R_2=\begin{bmatrix}1.1615 & 0.3468\\ 0.3468 & 1.5860\end{bmatrix},$$

$$R_3=\begin{bmatrix}3.4232 & -0.0316\\ -0.0316 & 0.9720\end{bmatrix}, R_4=\begin{bmatrix}3.4050 & -0.0138\\ -0.0138 & 0.9510\end{bmatrix},$$

$$N_1=\begin{bmatrix}0.8896 & -1.2674\\ -1.2674 & 1.8057\end{bmatrix}, N_2=\begin{bmatrix}0.4347 & -0.4957\\ -0.4957 & 0.7540\end{bmatrix},$$

$$K=\begin{bmatrix}13715 & 98\\ -11014 & 1906\end{bmatrix}, G=\begin{bmatrix}-1050.3 & -70.3\\ 823.1 & -983.4\end{bmatrix},$$

$$J^* = 1.2848$$

Figures 1-2 show the trajectories of states and their estimate errors for slow time-varying delay case. It is seen that the system is converged to the stable state.

B. A fast time-varying delay

In the case of a fast time-varying delay case: $h = 1.5$, $d = 1.1$, $\tau = 0.5$ we have the results as follows.

Similar with a slow time-varying delay case, we find

$$W_1=\begin{bmatrix}1.5612 & -1.5612\\ -1.5612 & 1.5612\end{bmatrix}, W_2=\begin{bmatrix}0.9888 & 0.3837\\ 0.3837 & 0.4773\end{bmatrix},$$

$$W_3=\begin{bmatrix}0.5000 & -0.5000\\ -0.5000 & 0.5000\end{bmatrix}, W_4=\begin{bmatrix}0.7071 & 0\\ 0 & 0\end{bmatrix},$$

$$W_5=\begin{bmatrix}0.8660 & -0.8660\\ -0.8660 & 0.8660\end{bmatrix}, W_6=\begin{bmatrix}1.2247 & 0\\ 0 & 0\end{bmatrix}$$

and

$$P_1=\begin{bmatrix}12.1578 & -5.7379\\ -5.7379 & 10.2535\end{bmatrix}, P_2=\begin{bmatrix}2.2573 & -0.8316\\ -0.8316 & 0.8316\end{bmatrix},$$

$$P_3=\begin{bmatrix}2.8013 & -1.9136\\ -1.9136 & 2.1666\end{bmatrix}, P_4=\begin{bmatrix}5.0991 & -0.9120\\ -0.9120 & 2.6311\end{bmatrix},$$

$$R_1=\begin{bmatrix}1.3938 & -0.1819\\ -0.1819 & 2.4175\end{bmatrix}, R_2=\begin{bmatrix}0.7803 & 0.3542\\ 0.3542 & 0.9604\end{bmatrix},$$

$$R_3=\begin{bmatrix}3.1408 & -0.2319\\ -0.2319 & 0.5347\end{bmatrix}, R_4=\begin{bmatrix}5.1500 & -2.0006\\ -2.0006 & 1.9634\end{bmatrix},$$

$$N_1=\begin{bmatrix}1.1510 & -0.8790\\ -0.8790 & 0.9409\end{bmatrix}, N_2=\begin{bmatrix}1.0886 & -0.6534\\ -0.6534 & 0.6650\end{bmatrix},$$

$$K=\begin{bmatrix}-21596 & -6067\\ 30936 & -587.1\end{bmatrix}, G=\begin{bmatrix}12904 & 778.5\\ -5561.8 & 2850.9\end{bmatrix},$$

$$J^* = 1.6499$$

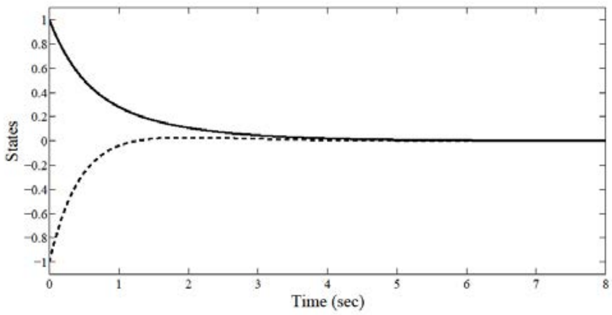

Fig. 1. Trajectories of states x_1 (—) and x_2 (- -).

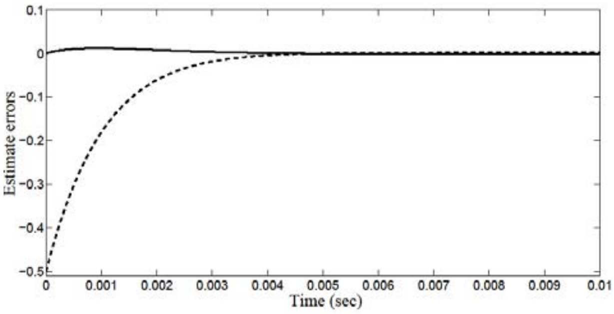

Fig. 2. Trajectories of estimate errors e_1 (—) and e_2 (- -).

V. Conclusion

This paper discusses a guaranteed cost filter design for uncertain neutral systems with time-varying delay. A sufficient condition for the guaranteed cost filter is derived on the basis of the LMI feasible solutions. The optimal cost is provided by minimizing the upper bound of the guaranteed cost. A numerical example is given to illustrate the proposed method.

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