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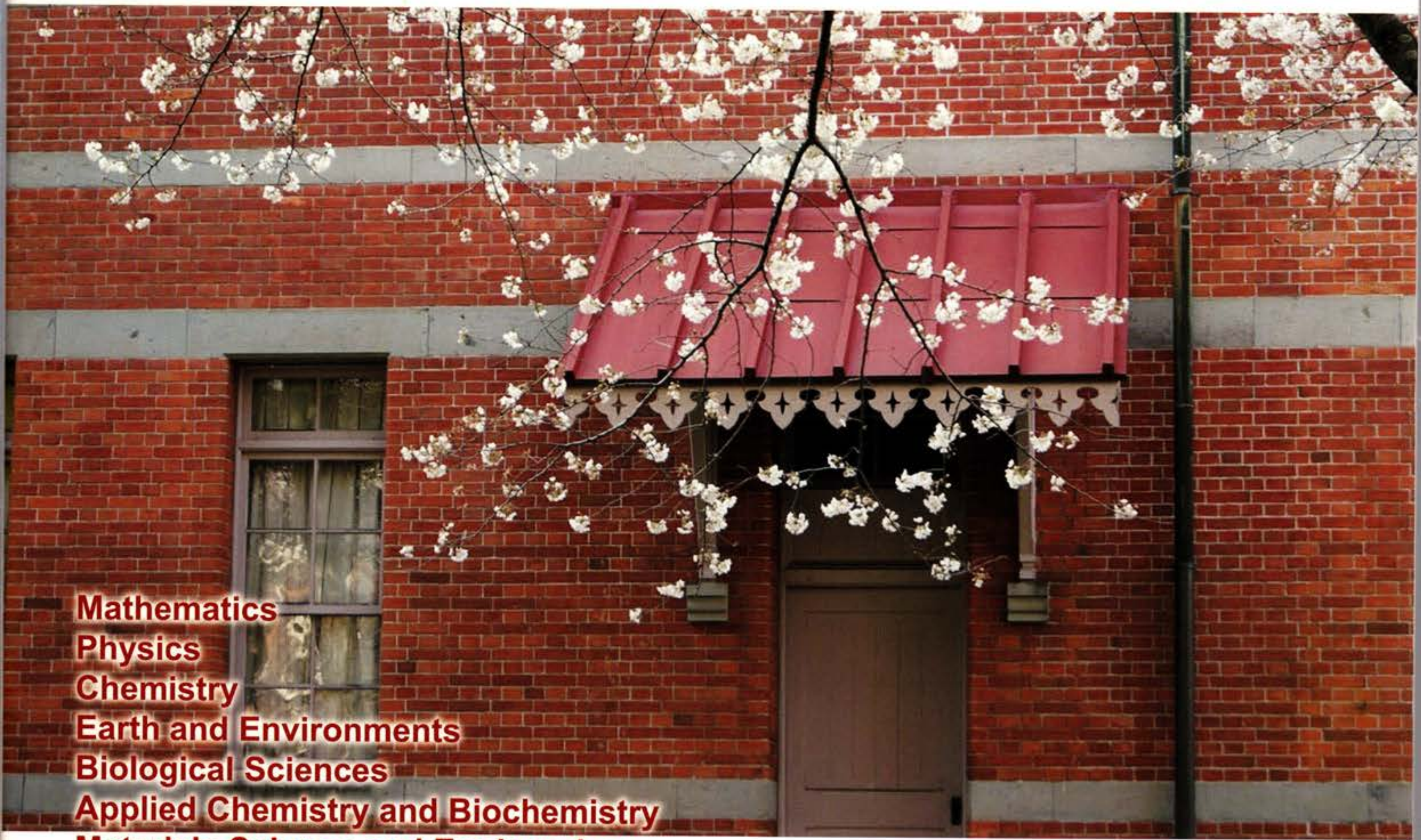

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A minimal order observer-based guaranteed cost control for systems with uncertainty in the state matrix

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Abstract: This paper studies a design problem of a minimal order observer-based guaranteed cost controller which not only achieves the stability but also guarantees an adequate level of performance. A sufficient condition for the guaranteed cost problem is given as a matrix inequalities optimization problem which is solved by use of a toolbox of linear matrix inequalities. A numerical example is shown to confirm the proposed method.

Index Terms-: robust control, guaranteed cost control, minimal order observer

I. INTRODUCTION

Considerable attention to the stability and stabilization problem for uncertain systems has been increasing for several last decades. However, designing feedback control systems that guarantee stability and optimal performance is rather difficult. One approach to this problem is a guaranteed cost control method that guarantees not only the stability but also an adequate level of performance [1]. Generally, guaranteed cost controllers are given by state feedback. However, it may not be possible to obtain all state variables in practical sense [2]. Therefore, observer-based guaranteed cost control is preferable in this situation rather than pure state feedback control.

This paper investigates an observer-based guaranteed cost control problem and provides a sufficient condition on a matrix inequalities optimization problem that can be written in LMIs terms [3]. We treat a minimal order observer [4], whereas some of states are unknown, but their mean and covariance are known. Further, uncertainty is considered in the state matrix. A numerical example is shown to indicate the effectiveness of the proposed method.

II. PROBLEM STATEMENT

Consider a continuous-time uncertain system

$$\dot{\boldsymbol{x}}(t) = (A + \Delta A(t))\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{1}$$
$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{2}$$

where $\boldsymbol{x}(t) \in \Re^n$ is the state vector, $\boldsymbol{u}(t) \in \Re^r$ is the control input vector, $\boldsymbol{y}(t) \in \Re^m$ is the measured output vector, A, B, C, D_A, E_A are known constant real-valued matrices with appropriate dimensions, and C is restricted to the form of $C = [O \quad I_m]$. A matrix $\Delta A(t)$ denotes real-valued matrix functions representing parameter uncertainties. It is assumed that

$$\Delta A(t) = D_A F_A(t) E_A \tag{3}$$

with

$$F_A^T(t) F_A(t) \leq I$$

where D_A and E_A are constant real-valued known matrices. We also assume that the initial state variable $\boldsymbol{x}(0)$ is unknown, but their mean and covariance are known

$$E\left[\boldsymbol{x}(0)\right] = \boldsymbol{m}_0 \tag{4}$$
$$E\left[(\boldsymbol{x}(0) - \boldsymbol{m}_0)(\boldsymbol{x}(0) - \boldsymbol{m}_0)^T\right] = \Sigma_0 > O \tag{5}$$

The objective is to design a controller

$$\boldsymbol{u}(t) = K\hat{\boldsymbol{x}}(t) \tag{6}$$

and a minimal order observer

$$\dot{\boldsymbol{z}}(t) = D\boldsymbol{z}(t) + E\boldsymbol{y}(t) + F\boldsymbol{u}(t) \tag{7}$$
$$\hat{\boldsymbol{x}}(t) = P\boldsymbol{z}(t) + W\boldsymbol{y}(t) \tag{8}$$

with

$$D = A_{11} + LA_{21},\ PT + WC = I_n,$$
$$F = TB,\ TA - DT = EC,\ A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right],$$
$$P = \begin{bmatrix} I_{n-m} & \boldsymbol{0} \end{bmatrix}^T,\ T = \begin{bmatrix} I_{n-m} & L \end{bmatrix}$$

so as to achieve an upper bound on the following quadratic performance index

$$E\left[J\right] = E\left[\int_0^\infty (\boldsymbol{x}^T(t)Q\boldsymbol{x}(t) + \boldsymbol{u}^T(t)R\boldsymbol{u}(t))dt\right] \tag{9}$$

associated with the uncertain system (1) and (2) where Q and R are given symmetric positive-definite matrices.

III. MAIN RESULTS

In this section, it is assumed that the feedback gain matrix has a form of

$$K = -R^{-1}B^T S_1 \tag{10}$$

where S_1 is a symmetric positive-definite matrix.

The main result of this study is given by Theorem.

Theorem. If the following matrix inequalities optimization problem; min $\{\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4\}$ subject to

$$\begin{bmatrix} \Lambda_0 & XE_A^T & XE_A^T & X^T \\ E_A X & -\zeta I & 0 & 0 \\ E_A X & 0 & -\theta I & 0 \\ X & 0 & 0 & -Q^{-1} \end{bmatrix} < 0 \qquad (11)$$

$$\begin{bmatrix} \bar{\Lambda}_0 & ZD_A & P^T S_1 B \\ D_A^T Z^T & -\theta_{inv} I & 0 \\ B^T S_1 P & 0 & -R \end{bmatrix} < 0 \qquad (12)$$

$$\sum_{k=1}^{n} \boldsymbol{e}_{nk}^T \Theta_0 \boldsymbol{e}_{nk} < \gamma_0, \quad \sum_{k=1}^{m} \boldsymbol{e}_{mk}^T \Theta_1 \boldsymbol{e}_{mk} < \gamma_1$$

$$\sum_{k=1}^{m} \boldsymbol{e}_{mk}^T \Theta_2 \boldsymbol{e}_{mk} < \gamma_2, \quad \sum_{k=1}^{m} \boldsymbol{e}_{mk}^T \Theta_3 \boldsymbol{e}_{mk} < \gamma_3 \qquad (13)$$

$$\begin{bmatrix} -\gamma_4 & \boldsymbol{v}_1^T Y^T & \boldsymbol{v}_2^T Y^T & \cdots & \boldsymbol{v}_m^T Y^T \\ Y\boldsymbol{v}_1 & -S_2 & & & \vdots \\ Y\boldsymbol{v}_2 & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ Y\boldsymbol{v}_m & \cdots & \cdots & \cdots & -S_2 \end{bmatrix} < 0 \qquad (14)$$

where

$$\Lambda_0 = AX + XA^T - BR^{-1}B^T + \zeta D_A D_A^T$$

$$\bar{\Lambda}_0 = S_2 A_{11} + A_{11}^T S_2 + Y A_{21} + A_{21}^T Y^T$$

$$Y = S_2 L, \ Z = [S_2 \ \ Y],$$

$$\Theta_0 = \frac{1}{2}(S_1(\Sigma_0 + \boldsymbol{m}_0 \boldsymbol{m}_0^T) + (\Sigma_0 + \boldsymbol{m}_0 \boldsymbol{m}_0^T)^T S_1)$$

$$\Theta_1 = \frac{1}{2}(S_2 \Sigma_{11} + \Sigma_{11} S_2), \ \Theta_2 = \frac{1}{2}(Y \Sigma_{21} + \Sigma_{21}^T Y^T)$$

$$\Theta_3 = \frac{1}{2}(Y^T \Sigma_{12} + \Sigma_{12}^T Y), \ \Sigma_{22}^{1/2} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_m]$$

$$\Sigma_0 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}, \ \boldsymbol{e}_{ik} = \begin{bmatrix} \boldsymbol{0}_{k-1}^T & 1 & \boldsymbol{0}_{i-k}^T \end{bmatrix}^T$$

has a solution $S_1 > 0$, $S_2 > 0$, $X > 0$, Y, Z, $\zeta > 0$, $\theta > 0$, $\theta_{inv} > 0$, γ_0, γ_1, γ_2, γ_3, γ_4 which satisfy the relation $\theta^{-1} = \theta_{inv}$, and $S_1^{-1} = X$, then the minimal order observer-based control law (6)-(8) with (10) is a guaranteed cost controller which gives the minimum expected value of the guaranteed cost

$$E[J^*] = E\left[\boldsymbol{x}^T(0) S_1 \boldsymbol{x}(0) + \boldsymbol{\xi}^T(0) S_2 \boldsymbol{\xi}(0)\right] \qquad (15)$$

It is noted that the matrix inequalities (11)-(14) can be solved by an iterative linear matrix inequalities approach.

IV. AN NUMERICAL EXAMPLE

Consider a system with

$$A = \begin{bmatrix} -3 & 0 & -2 & 0 \\ 0 & -2 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 3 \\ 2 \\ -6 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} O_2 & I_2 \end{bmatrix}, \ \boldsymbol{m}_0 = \boldsymbol{0}_4, \ \Sigma_0 = I_4, \ R = 9,$$

$$Q = \mathrm{diag}(7, 15, 1, 3), \ D_A = \begin{bmatrix} 0.1I_2 & O_2 \\ O_2 & O_2 \end{bmatrix},$$

$$E_A = \begin{bmatrix} 0.3I_2 & 0.3I_2 \\ O_2 & O_2 \end{bmatrix}.$$

With LMI toolbox of Matlab, we obtain a solution

$$S_1 = \begin{bmatrix} 1.0230 & 0.0071 & 0.0178 & 0.0000 \\ 0.0071 & 4.0863 & 1.0341 & 1.0316 \\ 0.0178 & 1.0341 & 1.0261 & 1.0111 \\ 0.0000 & 1.0316 & 1.0111 & 7.0391 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.3307 & -0.3356 & 0.3360 & -0.3373 \end{bmatrix},$$

$$E[J^*] = 13.5788.$$

A trajectory of the guaranteed cost $\gamma = \gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4$ is depicted in Fig.1

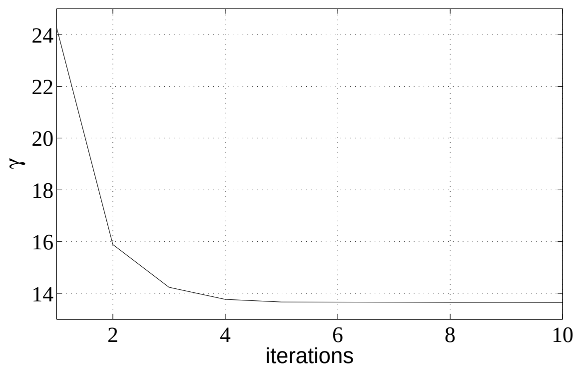

Fig. 1: Trajectory of γ.

V. CONCLUSION

A sufficient condition for the existence of a minimal order observer-based guaranteed cost controller has been established. To illustrate the proposed method, a numerical example is given. Extension to the time delay systems is remained for a future study.

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